TE 302

DISCRETE SIGNALS AND SYSTEMS

- Study on the behavior and processing of information bearing functions as they are currently used in human communication and the systems involved.
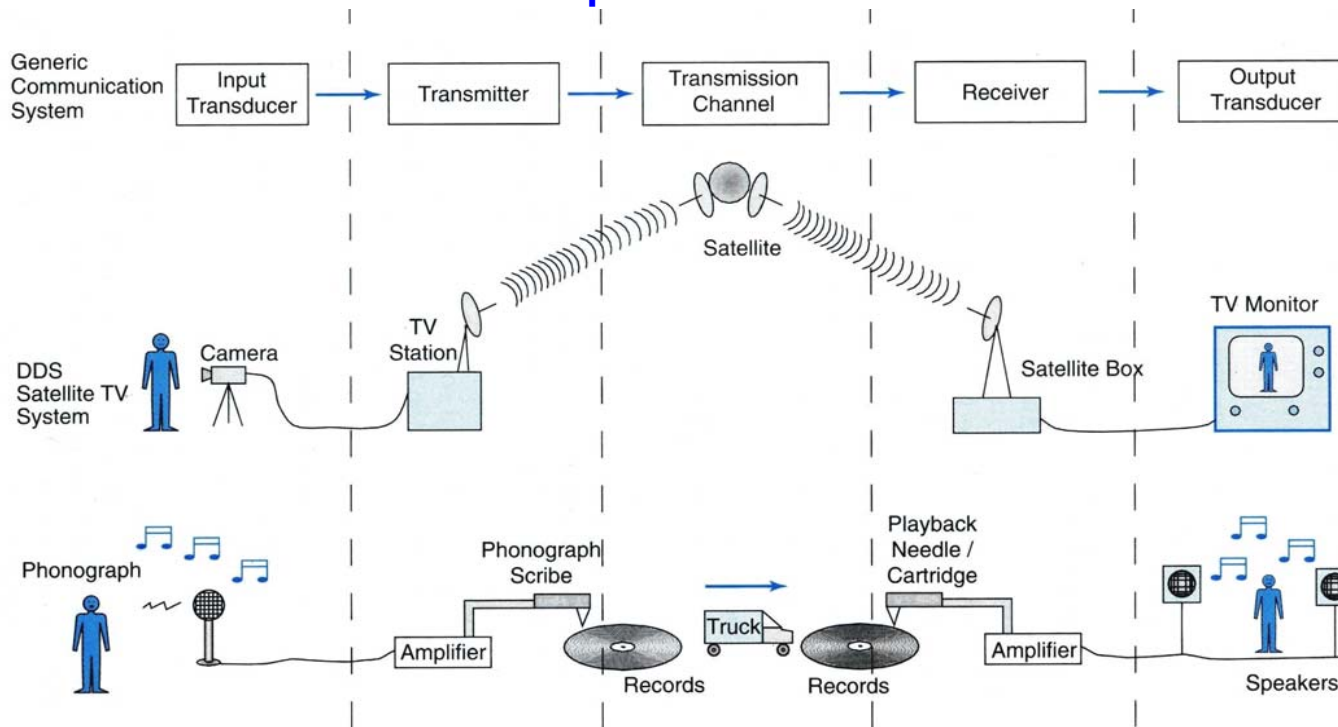
Chapter 1: INTRODUCTION

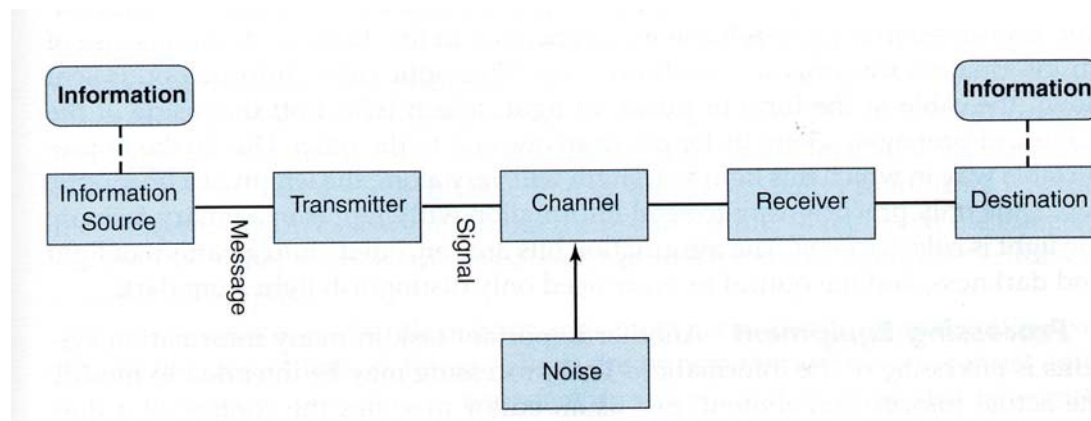Figure 1.1 Block diagram of a generic communication system.

Figure 1.2 End-to-End Single User Communication System Model.

Information: Set of symbols generated by a person or a system that wants to send them across a transmission medium to another person or a system through electronic means.

    Examples:

      Speech/audio signal; Image/video feeds; telemetry and other sensor data;
      Computer bits and words.

Information Source: To transmit information using electronic means must convert them into a set of electrical signals.

    Example:

      CCD camera is used for converting optical visual messages into video signals. This step is called the data acquisition process. It is also known as the transducer in classical communication electronics terminology.

Signal: $x(t)$ Broadly defined as any physical quantity that varies as a function of time and/or space and has the ability to convey information.

    - x represents the dependent variable (Voltage, picture intensity, pressure, etc.)
    - t the represents the independent variable (Time, space for imagery, etc.).

We can classify signals into four categories according to their appearance in the time-domain.

  (i) Analog or continuous signals

(ii) Discrete-time, (the amplitudes could be non-discrete or discrete)

(iii) Discrete-amplitude, where the time-stamp could be continuous or discrete.

(iv) Digital. Both input and the output signals are completely discretized and most often mapped into a sequence of binary numbers $\{0,1\}$.

System: A physical entity that operates on a set of primary signals (the inputs) to produce a corresponding set of resultant signals (the outputs). Operations, or processing, may take several forms: modification, combination, decomposition, filtering, extraction of parameters, etc.

System characterization: Can be represented mathematically as a transformation between two signal sets:

$$x[n] \in S_1 \quad \Rightarrow \quad y[n] = T\{x[n]\} \in S_2$$

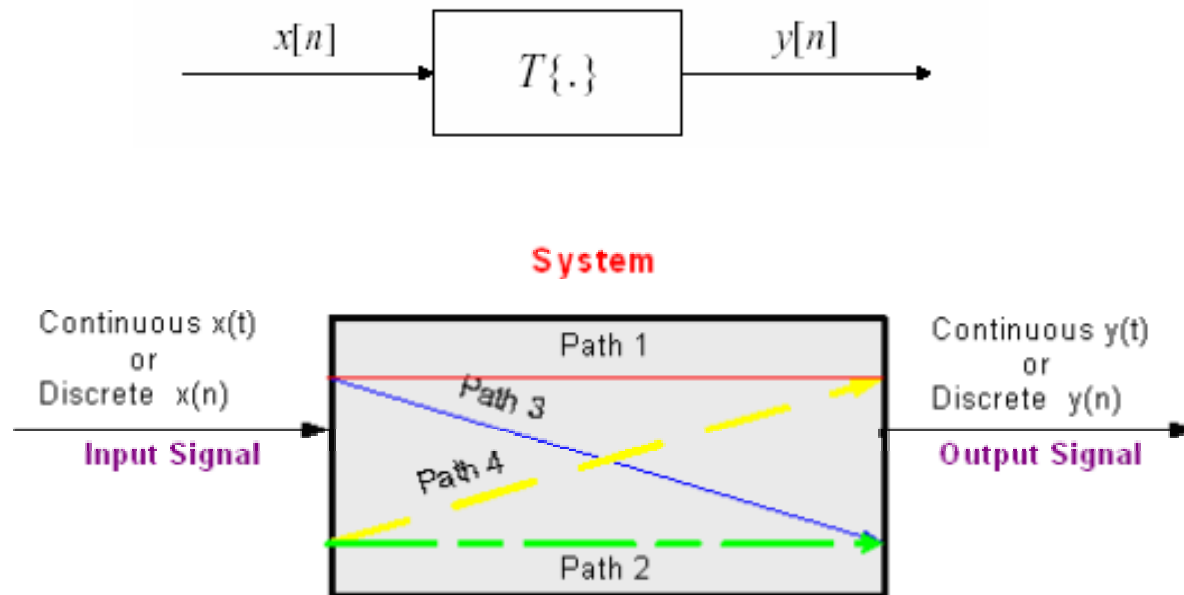where $T\{\bullet\}$ is a mapping rule between the input and output ports of the system as illustrated in Figure 1.3.

Figure 1.3 Generic System Model and Classifications.
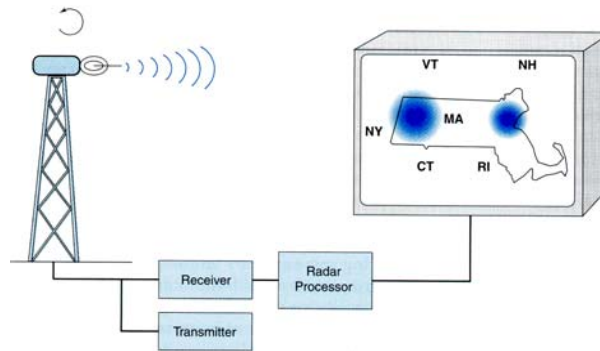
Example: Classical Radar System:

Figure 1.4 Classical radar systems

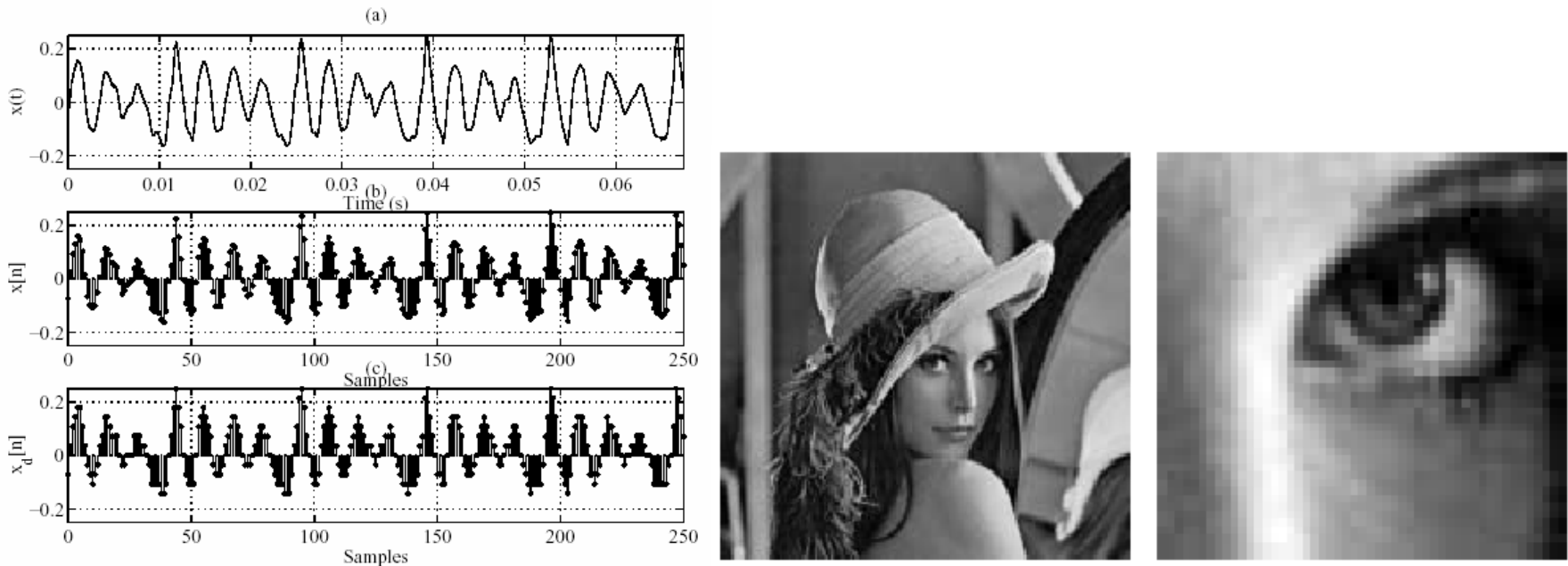**Example:** Speech sample or an image frame of Figure 1.5

Figure 1.5 (a) Recording of a vowel "a" sound as it was picked up by the mike, sampled (discretized) & Digital forms sampled at 4.0 kHz and quantized into 16 levels (4-bits). (b) Digitized image "Lena" (8-bits=256 levels and zoom into the left eye.

Example: Wavelets on a computer bus during a data transfer operation between a harddisk subsystem and the CPU as .shown in Figure 1.6
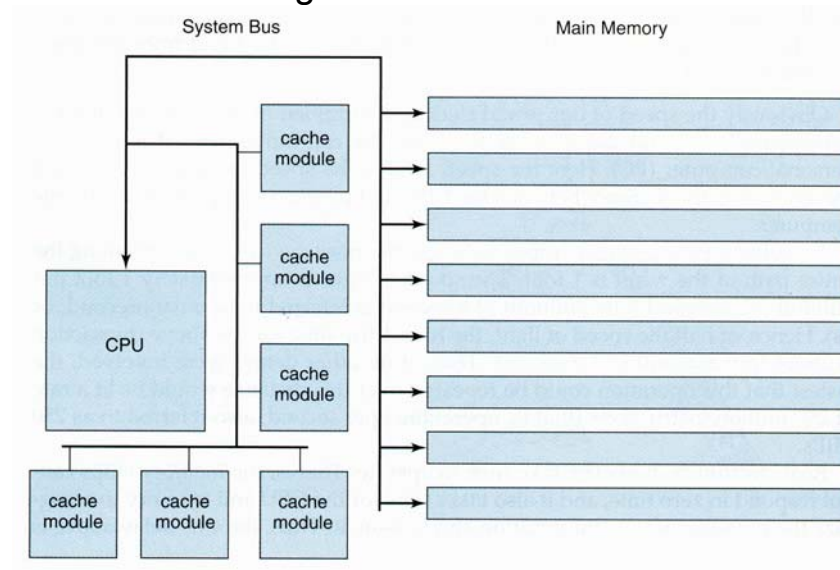

Figure 1.6 Block diagram of a PC motherboard.

Transmitter: Except public announcement (PA) systems or audio equipment in meetings where the distance between the source and all the listeners is very short, signals in nature cannot be transmitted in an electrical or any other communication medium.

For instance, audio signals of an FM100 radio station are shifted into the vicinity of 100 MHz with a frequency spread of 200 kHz. That is, the frequency allocation of this station is between 99.9 MHz and 100.1 MHz. The equipment used for shifting the audio signal to this frequency range is called the transmitter.

Channel: In communication theory and systems terminology, the transmission medium where the signals are transmitted is called *"communication channel."* Examples:
- Twisted-pair of wires,
- Coaxial cables,
- Waveguides,
- Fiber-optic links, and
- Radio waves.

As the signals traverse in a channel from a transmitter to a receiver, they attenuate according to the propagation laws of physics. In some cases, the attenuation is not extreme and the signal is picked up easily by an antenna of a receiver or by the I/O port of the data communication system. When distances are long, then a set of repeaters are used for detecting, amplifying and re-transmitting the signal.

Example: if one wants to send TV signals from North America to Australia, the signals are uploaded to a satellite nearest to the transmitter then the information hops from a satellite to another satellite until it is seen by the particular Australian TV station.

Distortion, Interference, and Noise: In addition to attenuation, signals during transmission are faced with a number of other ills. In the engineering jargon, all forms of degradation are loosely called **noise**. By changing the shape and characteristics of a given signal, noise limits the ability of the intended receiver to make correct symbol decisions, and thereby, it effects the rate of reliable communication.

Receiver: Attempts to undo the modifications made at the transmitter and combat degradations took place in the channel: distortion, intersymbol and interchannel interferences, echoe, and noise.

Destination: Received electrical signal is converted to a message form, mostly the original input message category sent by the source to the user, replica of the speaker's voice in the telephone set of the user.

User Information: Intended user of the input information-bearing message is usually a replica of the original message. However, the message coming to a user may not be an identical replica of the sender's information. The access control of a safe room by voiceprint of a legitimate/imposter user is an example to this type of communication. At the end of the verification process from a given voiceprint, either he/she is given the access or denied. In other words, a single bit instruction to the door lock is the final message.

1.1 Signal Classifications

1.1.1 Deterministic and Random Signals:

If a signal is known at all times either explicitly by a mathematical formula or by an array of numbers then it is deterministic signal.

Example: Sinusoidal waveform:

$$x(t) = A.Cos(2\pi f_0.t) \tag{1.1}$$

where A is the amplitude and f_0 is the fundamental frequency of the sinusoidal waveform generator and t is the independent time variable. In (1.1), the value of $x(t)$ is exactly known at all times.

if the sinusoidal waveform generator had a random phase component:

$$x(t) = A.Cos(2\pi f_0.t + \theta_0) \tag{1.2}$$

Here θ_0 is the initial random phase of the generator, which could equally likely be anything in the range $-\pi < \theta_0 \leq \pi$. This type of signals is called random signals since we cannot be exactly sure of their values at a given time.

Example: All forms of interferences in the cockpit of an airplane, such as, the engine noise, wind noise and the interference of other persons' conversation while the pilot is getting landing instructions from the tower. In this example, the engine noise and the wind noise could be considered natural signals, whereas, the interference from other people is a man-made noise.

- In the context of communication systems, the study of random signals, or noise, is very important.

- Deterministic signals are useful for understanding the inner workings of systems engineering problems.

1.1.2 Frequency Band Classification:

This classification is done according to the frequency band signals occupy. When we study the frequency content of signals we notice that they are always spread over a finite band of frequencies.

Example: Telephone speech communication: The upper frequency is in the neighborhood of 3,300-3,600 Hz. Understandably, for other signals the range will be different. These bands are given different names:
- Extremely low frequency band (ELF) used for underwater acoustics signals (under 300 Hz).
- Voice frequency band of telephony and audio signals (300 Hz - 30 kHz).
- AM signals are used in radio communication based on amplitude modulation (500 KHz - 1.6 MHz).
- VHF signals for TV broadcasting (30 MHz - 300 MHz)
- FM band for radio communication based on frequency modulation (88 MHz - 108 MHz), and
- UHF (300 MHz - 3.0 GHz) for video, data and telemetry signals.

In addition, we have visible light and electromagnetic waves at the high frequency end of the frequency spectrum used in various applications. In all of these signals there is one underlying question:

Is the signal occupying a particular frequency band when it occurs in the nature? Equivalently, Is it translated to that specific frequency range from a lower one by a system?

1.1.3 Baseband Passband Signal Classification:

- If the frequency range where the signal is generated in the nature then is called a baseband signal.
- If a signal is translated to another band of frequencies by a man-made system or a natural system then we have a passband signal.

Example: Speech coming out of a public announcement system is a baseband signal, whereas, the signal transmitted by an FM transmitting antenna is a frequency modulated passband signal obtained by heterodyning (mixing) the audio signal with a carrier frequency.

1.1.4: Energy and Power Signals:

- Signals can be also classified according to their energy and power contents. If a signal has a finite energy then it is called an energy signal.
- If it attains a finite power value then it is classified as a power signal.

Signals used in communication systems are all power signals, as it will be discussed in the next chapter.

1.2 Sampling Analog Signals (A/D and D/A Conversion)

Sampling process provides the link between a continuous (analog) signal and its discrete version. One of the classical examples of discrete-time signals is the sampled outputs of a telephone speech signal passing through an analog-to-digital (A/D) converter, where the speech waveform $x(t)$ is has peak amplitude levels in the range $\{-m_P \le x(t) \le +m_P\}$ as shown in Figure 1.7. Let us measure $x(t)$ at each clock instant T_0 :

$$x[n] = X[nT_0] = x(t)\big|_{t = nT_0} \quad for\ n = 0,1,2,\cdots \tag{1.3}$$

and call these output values x[n] samples of $x(t)$. This process is commonly known as the Pulse Amplitude Modulation (PAM) in the literature.
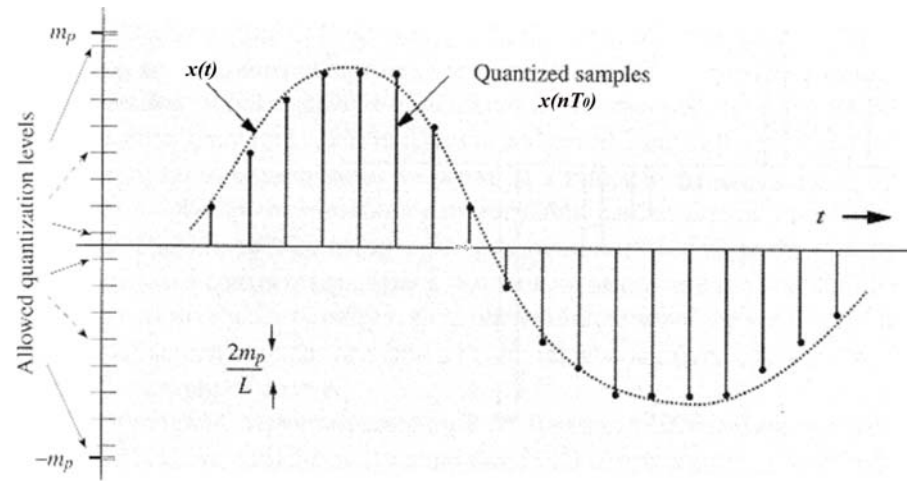
Figure 1.7 Quantization by Pulse Amplitude Modulation (PAM) Signals

Sampling (Nyquist) Theorem:

1. A/D Stage: If a continuous-time (analog) signal $x(t)$ has no frequency components (harmonics) at values greater than a frequency value f_{max} then this signal can be UNIQUELY represented by its equally spaced samples if the sampling frequency F_S is greater than or equal to $2f_{max}$. This is the analog-to-digital (A/D) conversion at F_S samples per second. A/D conversion stage.

2. D/A Stage: Furthermore, the original analog signal $x(t)$ can be TOTALLY recovered from its samples $x(n)$ after passing them through an ideal integrator (ideal low-pass filter) with an appropriate bandwidth. D/A Conversion Stage.

During the digital-to-analog converter stage, the signal is converted back to analog domain and this is commonly known as the synthesis or the reconstruction step. In speech, video, and computer communication tasks, the synthesized signal is desired to be a close replica of the original signal[1].

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[1] The minimum acceptable sampling frequency $F_S = 2.f_{max}$ is known as the NYQUIST RATE in the literature and the communication systems terminology. Real-life signals are always pre-filtered to $f_{max} \leq F_S/2$ before A/D Conversion stage to avoid a form of distortion known as the aliasing noise or spectral fold-over distortion.

Example: Impact of sampling rate on signals is be illustrated in Figure 1.8, where the first 5.0 ms segment of the sound "I" in the word "Information" spoken by a male speaker. Upper left is the original analog signal and the rest are the versions sampled at 1.0 KHz, 5.0 kHz, and 10.0 kHz. First two sampling rates have poor tracking capability of the original. But he last figure has an envelope very close to that of the analog version.
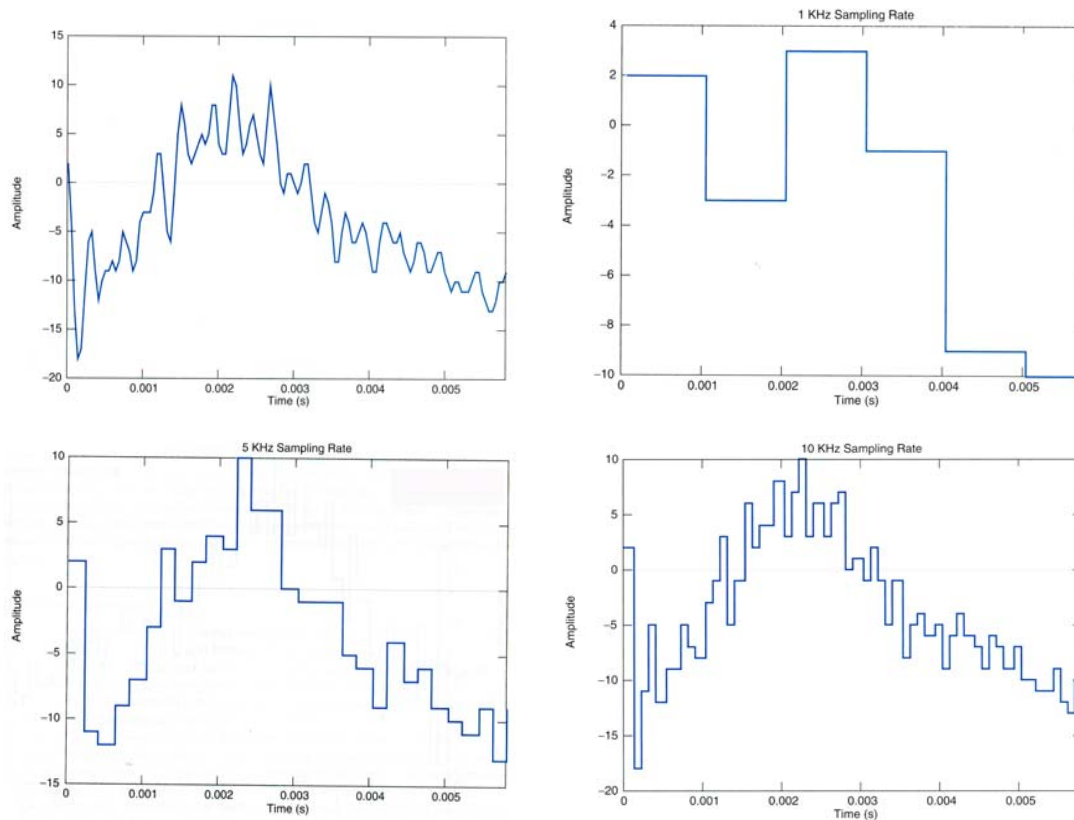

Figure 1.8 Effect of sampling rate on speech signals.

Example: Let us sample a 2Hz Sine wave at 1.6Hz, 3.2Hz and 6.4Hz over a 5 second interval (A/D Stage). In the reconstruction process (D/A Stage), the signal is then synthesized at the sample resolution which clearly shows the loss of signal information when a signal is undersampled.

Case 1: Sampling at 1.6 Hz. The number of samples is actually too small to be able to exactly recover the 2.0 Hz signal. It can also be seen that not all cycles of the signal are sampled, thereby resulting in the loss of signal information.
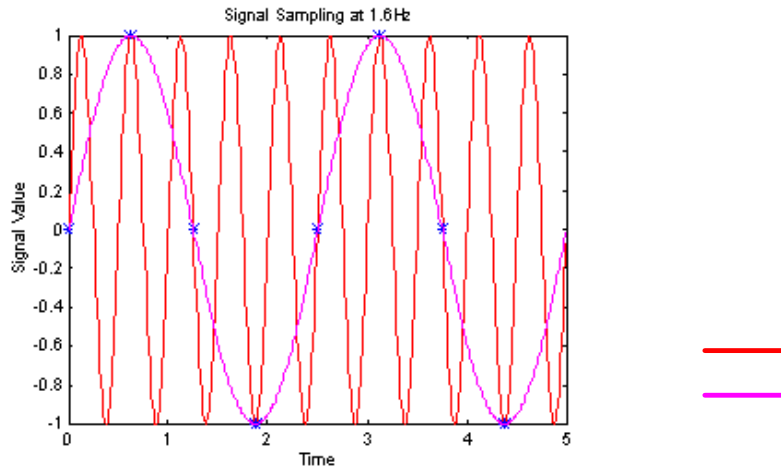
Case 2: Sampling at 3.2 Hz. Although the number of samples is larger than the previous case, we are still sampling below the Nyquist Rate. Thus there is some loss of information but it is not as severe as in the 0.8Hz case.
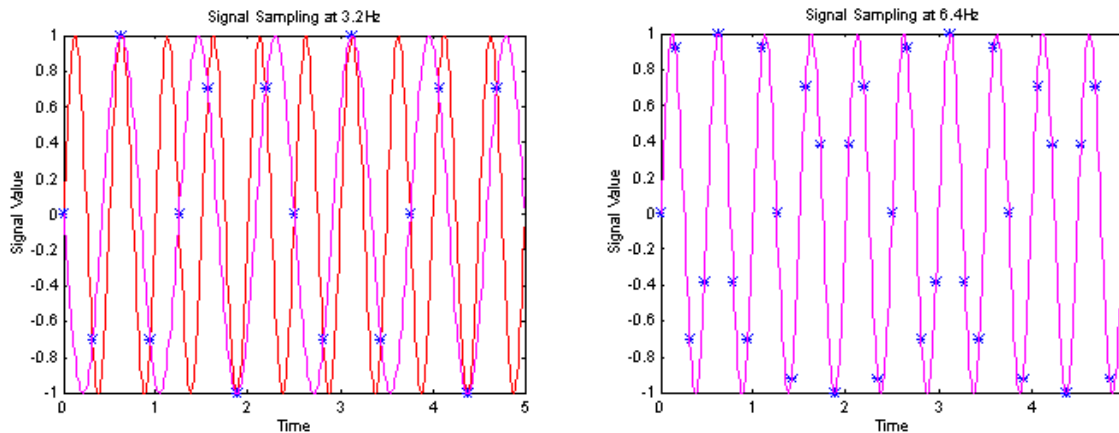
Case 3: Sampling at 6.4 Hz. Managed to reproduce the exact curve, original and the sampled version overlap exactly. There are more than two samples for each cycle of the signal. By increasing the sampling rate, we increase the number of samples which would result in a more accurately reproduced signal.

1.3 Quantization and Coding

- When an analog signal is sampled into a sequence of discrete signals with amplitudes limited to $\{-m_P, +-m_P\}$, we can easily turn them into a sequence of binary numbers (codewords) by dividing this range into a finite number of *L-levels* with a step size of $\Delta v \equiv 2m_P / L$.

- Next, we represent each interval by its center point. Since there are only *L* different center points we can assign one of *m-bits* long codewords to every center point. Therefore, we will need only $m = \log_2(L)$ bits of information for each sample.

- This step is called quantization and the overall process of sample and hold followed by binary mapping is known as the Pulse Code Modulation (PCM) in the community.

Example: Audio signal in Figure 1.9 is first sampled by an A/D converter and held at its sample values until they are transformed into a set of ones and zeros by a quantizer.
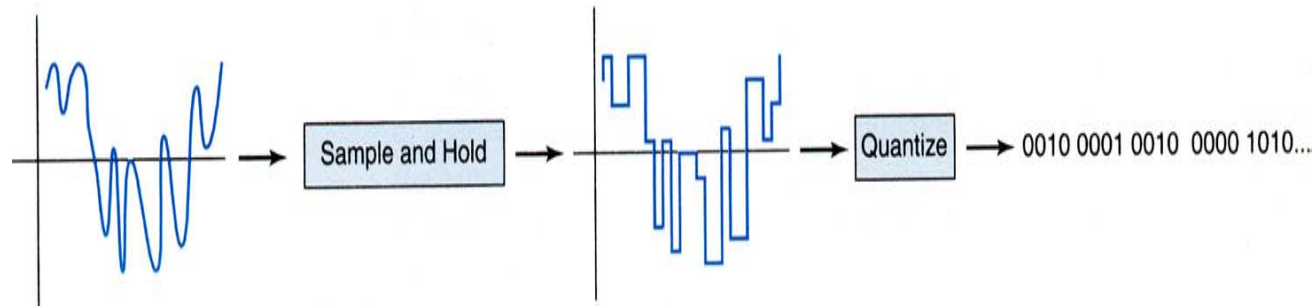

Figure 1.9 A simple audio sampling and quantization system.

Example: *m-ary* Signals: When we deal with integer only signals, such as, data transfer between a keyboard and the system we can use a 2-level pulse set. Suppose that our numbers range between 0 and 15. To transmit each of these 16 different characters we need a sequence of four (4) binary pulses. In other words, we can repeat the 2-level pulse shapes of Figure 1.9a four times according to binary representation of 16 numbers of Figure 1.10.
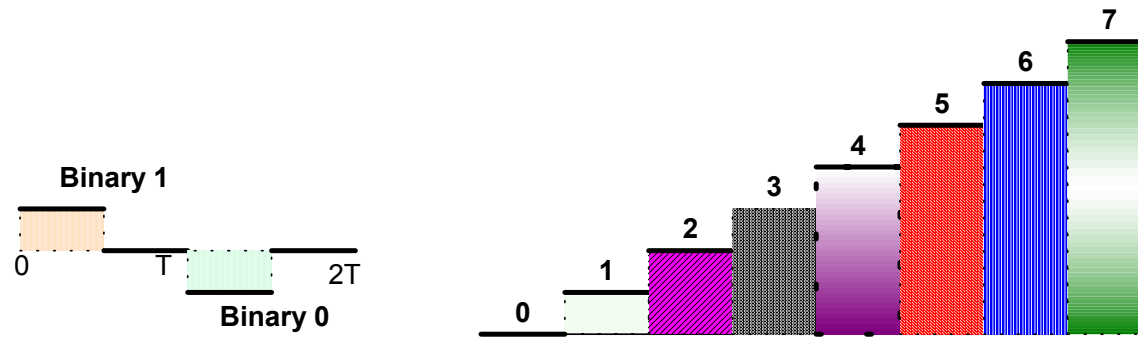
Figure 1.10 Multi-level Signaling. (a) 2-level pulse set; (b) 8-level pulse set.

However, if we use have 8-level octal pulse set of Figure 1.10 b then we will need two 8-level pulses per decimal number. Similarly, we can expand the octal set (8-level) into 16-level hexadecimal pulse set by flipping the shapes in Figure 1.10.b around the horizontal axis and adding a DC bias. These are tabulated in Figure 1.11.

Decimal	Octal	Hex Numeral	Bit Pattern
0	0	0	0000
1	1	1	0001
2	2	2	0010
3	3	3	0011
4	4	4	0100
5	5	5	0101
6	6	6	0110
7	7	7	0111
8	10	8	1000
9	11	9	1001
10	12	A	1010
11	13	B	1011
12	14	C	1100
13	15	D	1101
14	16	E	1110
15	17	F	1111

Figure 1.10 Decimal, Binary, Octal and Hexadecimal representation.