Chapter 6. ANGLE PULSE MODULATION SYSTEMS

We have discussed in Chapter 1 that the digital communication has significant advantages over its analog counterpart at the expense of increased bandwidth. However, the bandwidth cost is somewhat less of an issue with the explosion in broadband information transmission era. These days, even natural analog information such as speech, audio, radar, video, sonar and various signals from telemetry equipment and sensors are communicated using digital communication techniques. If we add these digitized signals on top of various digital data, it is not difficult to see that the volume of the traffic in information highways has become predominantly digital. In this chapter, we will start discussing how to encode continuous signals from analog sources into baseband pulse sequences and how to approximate them with digital bit streams, or equivalently quantized them. The basic block diagram governing this process is depicted in Figure 6.1.

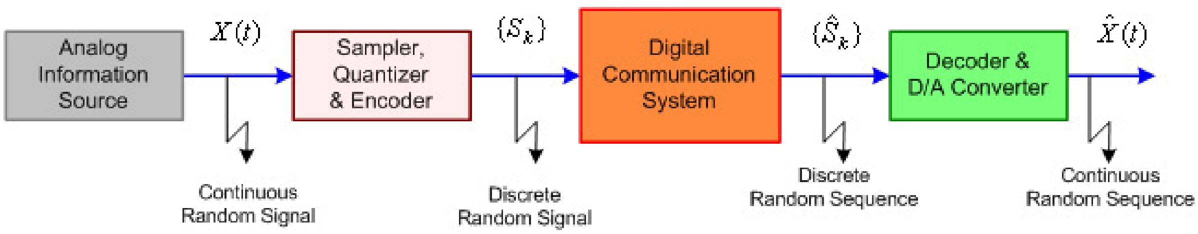


Figure 6.1 Transmission of analog signals using digital communication systems.

6.1 Sampling Theorem

Let us call the "Nyquist Sampling Theorem". If a continuous-time (analog) signal $x(t)$ has no frequency components (harmonics) at values greater than a frequency value, i.e. a bandwidth of $W\ Hz$ then this signal can be UNIQUELY represented by its equally spaced samples $\{S_k\}$ if the sampling frequency f_S is greater than or equal to $2W$. This is known as the analog-to-digital (A/D) conversion at a rate $R \geq 2W$ samples per second *(A/D conversion stage)*. Furthermore, the original analog signal $x(t)$ can be TOTALLY recovered from its samples $\{S_k\}$ after passing them through an ideal integrator (ideal low-pass filter) with an appropriate bandwidth *(D/A Conversion stage.)*

Note 1: Minimum acceptable sampling frequency $f_S = 2.W$ is known as the NYQUIST RATE in the literature and the communication systems terminology and period of sampling is given by $T_S = 1/f_S$.

Note 2: Before the A/D Conversion stage, signals in real-life are always band-limited to bandwidth $W \leq f_S/2$ to avoid a form of distortion known as the aliasing noise or spectral fold-over.

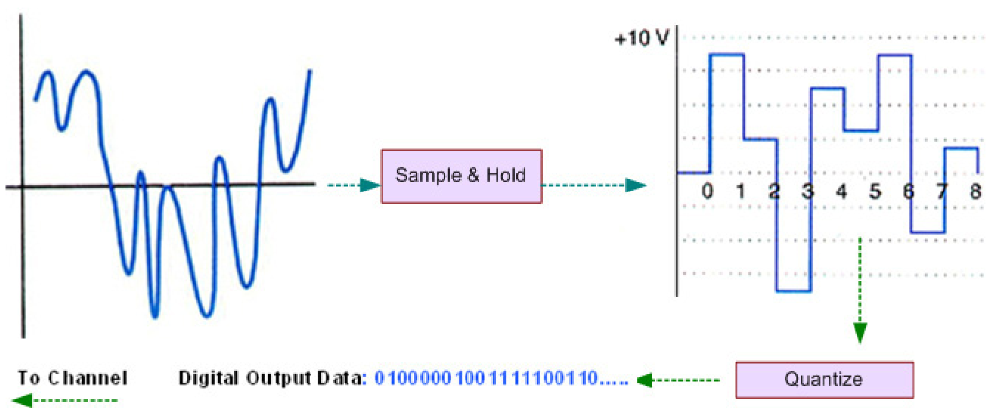


Figure 6.2. Sampling and quantization of analog signals.

The effect of sampling rate on analog signals is be illustrated in Figure 6.3, where the first 5.0 ms segment of the sound "I" in the word "Information" spoken by a male speaker. Upper left is the original analog signal and the rest are the versions sampled at 1.0 KHz, 5.0 kHz, and 10.0 kHz, respectively. It is to see that the first two sampling rates have poor tracking capability of the original, whereas, the last figure has an envelope very close to that of the analog version.

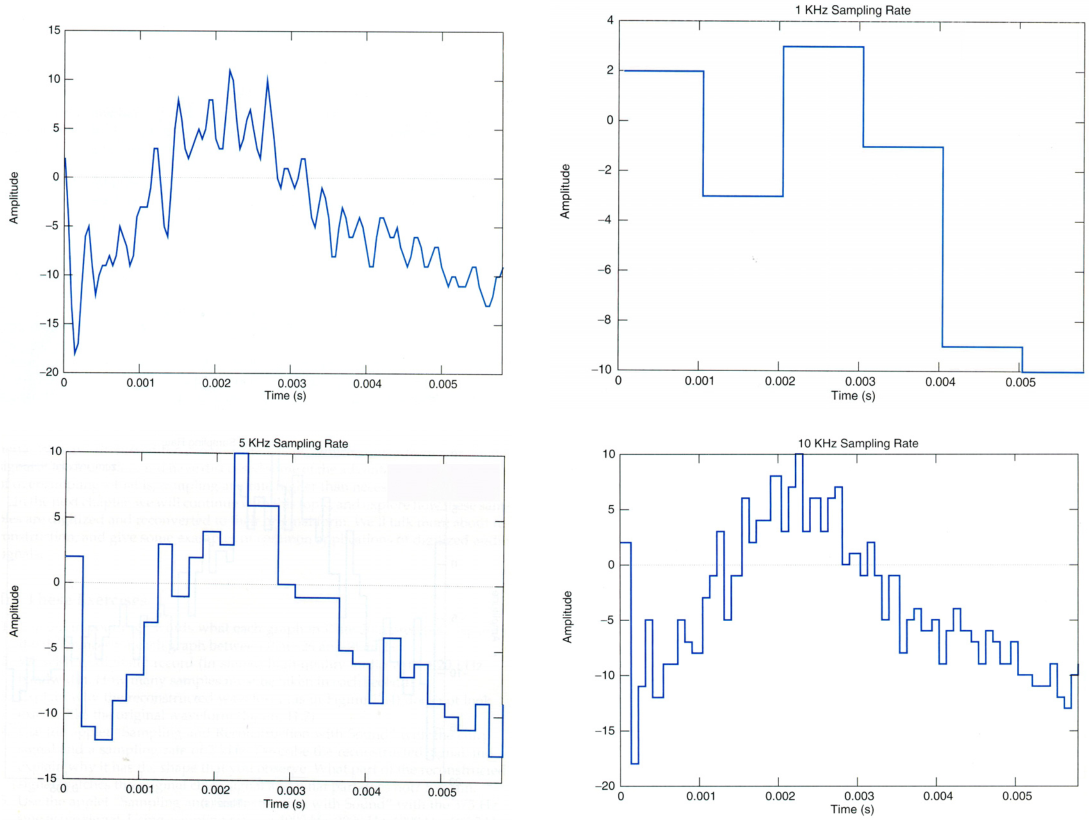


Figure 6.3 Effect of sampling rate on speech signals.

6.1.1 Sampling with a Switcher: An analog signal $x(t)$ have a bandwidth *W Hz,* or equivalently, $w_x = 2\pi W$ radians/second. We sample it not less than its Nyquist rate by multiplying with a sampling (switching) function, $s(t)$ and form a sequence of samples $x_S(t) = x(t).s(t)$ as illustrated in Figure 6.4.

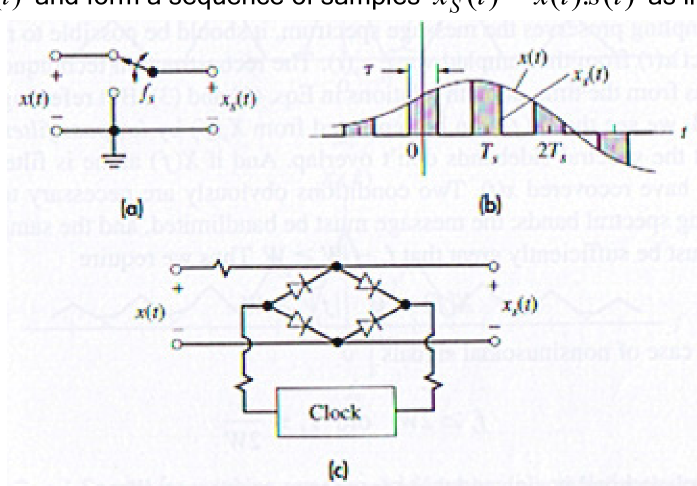


Figure 6.4 Sampling with a switcher (Carlson 6.1-1)

As we did in Chapter 4, let us write the Fourier cosine series of $s(t)$:

$$s(t) = C_0 + \sum_{n=1}^{\infty} 2.C_n.Cosnw_S t \tag{6.1}$$

which results in the sampled version of the signal:

$$\begin{aligned} x_S(t) &= x(t).C_0 + x(t).\sum_{n=1}^{\infty} 2.C_n.Cosnw_S t \\ &= C_0.x(t) + 2C_1.x(t).Cosw_S t + 2.C_2.x(t).Cos2w_S t + \cdots \end{aligned} \tag{6.2}$$

In the frequency-domain we have:

$$X_S(f) = C_0.X(f) + C_1.[X(f - f_S) + X(f + f_S)] + C_2.[X(f - 2f_S) + X(f + 2f_S)] + \cdots \tag{6.3}$$

The spectrum has the base-band spectrum scaled by C_0 as well as infinitely many scaled replica at multiples of the Nyquist frequency ash shown in Figure 6.5.

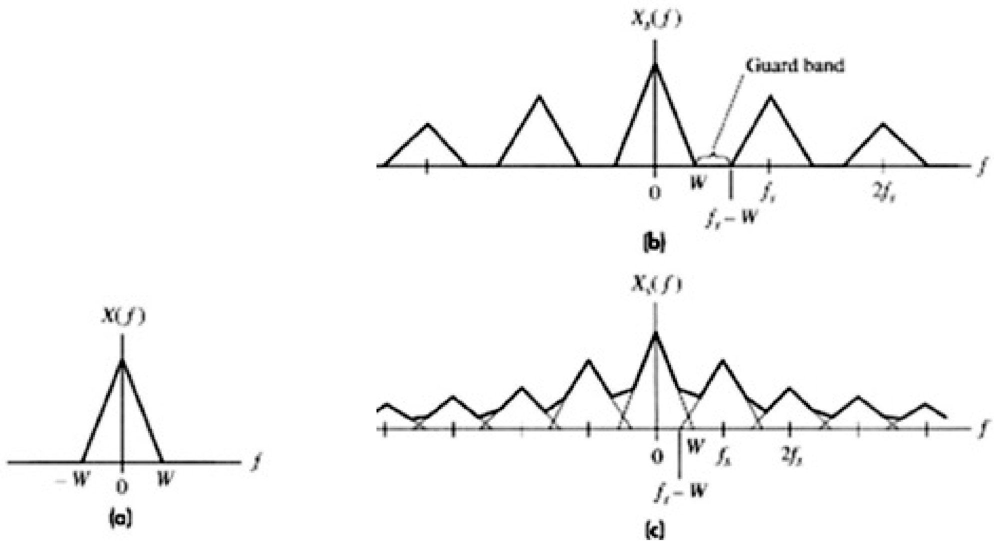


Figure 6.5 (a) Spectra for the baseband signal. (b) Spectrum of the sampled signal when $f_S > 2W$ and (c) Spectrum of the sampled signal when $f_S < 2W$ resulting in spectral fold-over.

Sampling requires special care, if the sampling frequency does not satisfy the Nyquist criterion then spectral fold-over takes place and the tails of the neighboring replica spectra overlaps with each other resulting in a non-recoverable distortion.

6.1.2 Ideal Sampling and Reconstruction (Interpolation):

In the case of ideal sampling, the switching function is an impulse train $\delta_{T_S}(t)$ with a sampling period of T_S. Hence, we can write it in the form:

$$x_\delta(t) = x(t).\delta_{T_S}(t) = x(t).\sum_k \delta(t - kT_S) = \sum_k x(kT_S).\delta(t - kT_S) \tag{6.4}$$

Frequency-domain equivalent of this result is shown below and in Figure 6.5. Again, spectral overlaps will be an issue if the Nyquist criterion is not satisfied.

$$X_\delta(f) = f_S.X(f) + f_S[X(f - f_S) + X(f + f_S)] + f_S[X(f - 2f_S) + X(f + 2f_S)] + \cdots \tag{6.5}$$

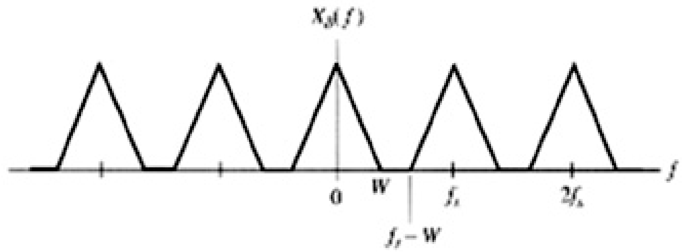


Figure 6.5 Spectrum of ideally sampled signal. (Carlson 6.1-5)

Reconstruction stage is commonly known as the D/A conversion and achieved by interpolating the discrete samples by use of an ideal Low-Pass filter with a bandwidth *B Hz*. This reconstruction process for the case in Figure 6.4 is pictorially shown in Figure 6.6.

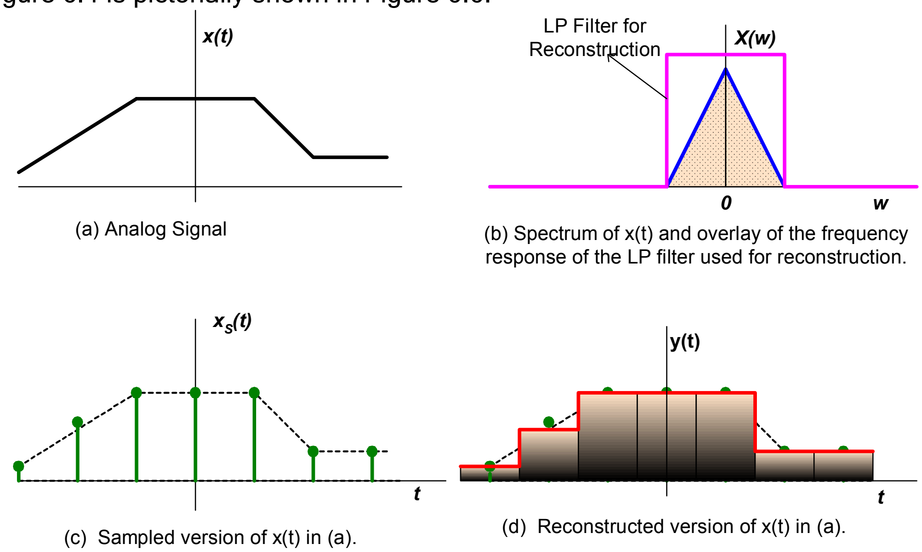


Figure 6.6 Reconstruction of the analog signal with and ideal LP filter.

Let us assume that the frequency response of this ideal LP-filter is given by:

$$H(f) = K.rect(\frac{f}{2B}).e^{-jwt_d} \tag{6.6}$$

where the filter gain is *K*, t_d the time delay, *B* the bandwidth. Inverse FT results in the impulse response:

$$h(t) = 2.K.B.Sinc2B(t - t_d) \tag{6.7}$$

Pushing the ideally sampled signal in (6.4 -6.5) through this filter will produce the shape in Figure 6.7.

$$Y(f) = H(f).X_\delta(f) = K.f_S.X(f).e^{-jwt_d} \tag{6.8a}$$

and

$$y(t) = F^{-1}\{Y(f)\} = K.f_S.x(t - t_d) \tag{6.8b}$$

Equivalently,

$$y(t) = h(t) * x_\delta(t) = \sum_k x(kT_S).h(t - kT_S)$$

$$= 2.B.K.\sum_k x(kT_S).Sinc2B(t - t_d - kT_S)$$

If we choose the filter bandwidth as $B = f_S / 2$, $K = 1.f_S$ and $t_d = 0$, we have:

$$y(t) = \sum_k x(kT_S).Sinc(f_S t - k) \tag{6.9}$$

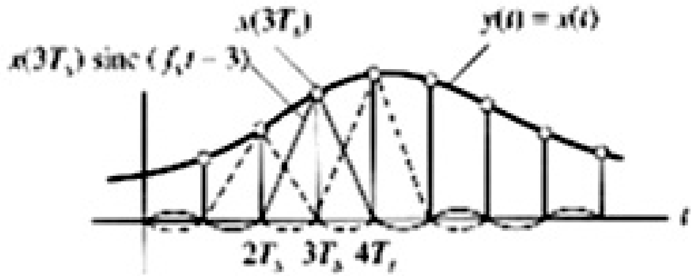


Figure 6.7 Ideal Reconstruction (Carlson 6.1-6)

Provided $f_S \geq 2W$ and the bandwidth *B* of the filter as constrained above, a bandlimited signal is completely recoverable from its samples. However, a filter requiring the perfect cancellation of high frequencies is not realizable since the impulse response obtained by the inverse Fourier transform of (6.7) will have infinitely long tails on both directions, leading to a non-causal system and, thus, a non-realizable filter. To remedy the situation, we use a modified approach using a Zero-Order Hold (ZOH), or a First-Order Hold (FOH) structures and more sophisticated scenarios.

Zero-Order Hold (ZOH) and First-Order Hold (FOH):

$$y(t) = \sum_k x(kT_S).rect(\frac{t-kT_S}{T_S}) \qquad y(t) = \sum_k x(kT_S).\Delta(\frac{t-kT_S}{T_S}) \qquad (6.10)$$

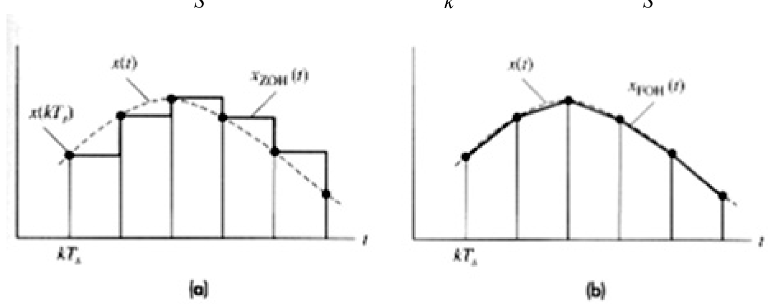


Figure 6.8 Signal Reconstruction using ZOH and FOH Approximations. (Carlson 6.1-8)

If we do not have all the higher order replicas eliminated by the practical filters we employ, aliasing is unavoidable the spectrum of the message will be corrupt as shown in Figure 6.9.

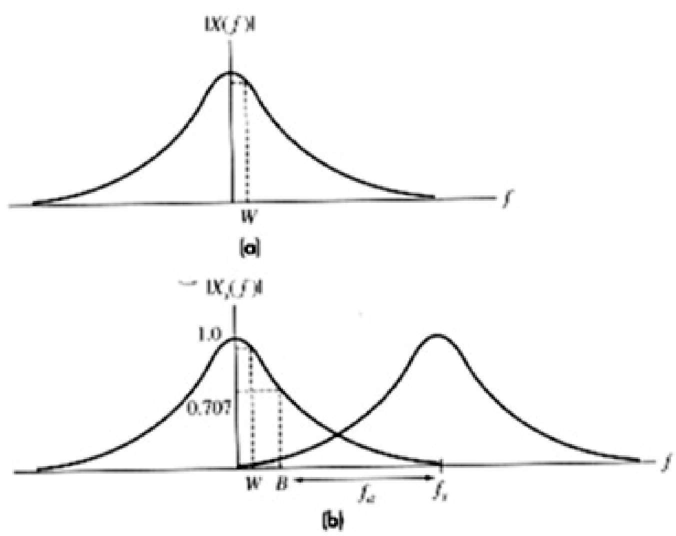


Figure 6.9 (a) Output of an RC filter used as a reconstruction filter; (b) After sampling. The segment between the origin and the frequency *B* represents the aliasing spillover the passband.

Shaded area (section between *W and B*) will the aliased (folded-over) components that have spilled into the passband region of the filter. However, this aliased area decreases if f_s increases or if we employ a filter with a larger bandwidth. Let us assume that we will be using a first-order Butterworth-type filter for reconstruction then the maximum aliasing error sneaking into the passband will be:

$$\% \ Error = (\frac{1/0.707}{\sqrt{1+(f_a/B)^2}}.100\% \qquad (6.11)$$

where $f_a = f_s - B$ and the 0.707 factor comes from the fact that we are using a first-order Butterworth filter, which is -3.0 dB down (0.707 of the peak value of amplitude) at its half-power.

Example 6.1: Let us assume that we implement of reconstruction LP filter with an RC network, where $R = 10\ k\Omega$ and $C = 100\ pF$. We like to have the power in the aliased components at least 30 dB down below the desired signal. What should be the desired sampling frequency for this signal to meet that requirement?

Recall that -3 dB corresponds ½ power point (50%) or (0.707 of the peak amplitude). Similarly, -30 dB would be 5% of the power.

$$B = \frac{1}{2\pi RC} = \frac{1}{2\pi.10^4.100.10^{-12}} = 159\ kHz$$

If we substitute this into (6.11):

$$5\% = \frac{1/0.707}{\sqrt{1+(f_a/159{,}000)^2}}.100\% \quad \Rightarrow \quad f_a = 4.49\ MHz$$

From the graph above we see that the sampling frequency must be

$$f_s = f_a + B = 4.49 + 0.159 = 4.65\ MHz.$$

Example 6.2: Let us assume that our signal is a single unit-impulse function: $x(t) = \delta(t)$. The sampled version will be:

$$x_S(t) = \begin{cases} 1 & t = 0 \\ 0 \quad x(\pm kT_S) & where\ k = 1,2,3,\cdots \end{cases} \qquad (6.12a)$$

and we sample at the Nyquist rate. When reconstructed we get:

$$y(t) = Sinc(2\pi Bt)\,. \qquad (6.12b)$$

It is worth noting that sampling at Nyquist rate does not leave room for gentler slopes on reconstruction. If we increase the sampling rate higher than this critical situation we have a case known as the over-sampling, which does not harm to the signal but the resulting at an increased transmission rate, i.e., bit rate increases as a result. On the other hand, if we sample below the Nyquist rate, then adjacent spectra overlap each other leading to the phenomenon known as the aliasing error (spectral folding).

In figure 6.10 we have an analog signal very similar to the case in Figure 6.6. As we can see from the sampled version in part c and the corresponding spectrum in part d, the signal representation is not very close. The LP reconstruction filter spans over the distorted common portions, which in time-domain results in a significantly different output signal

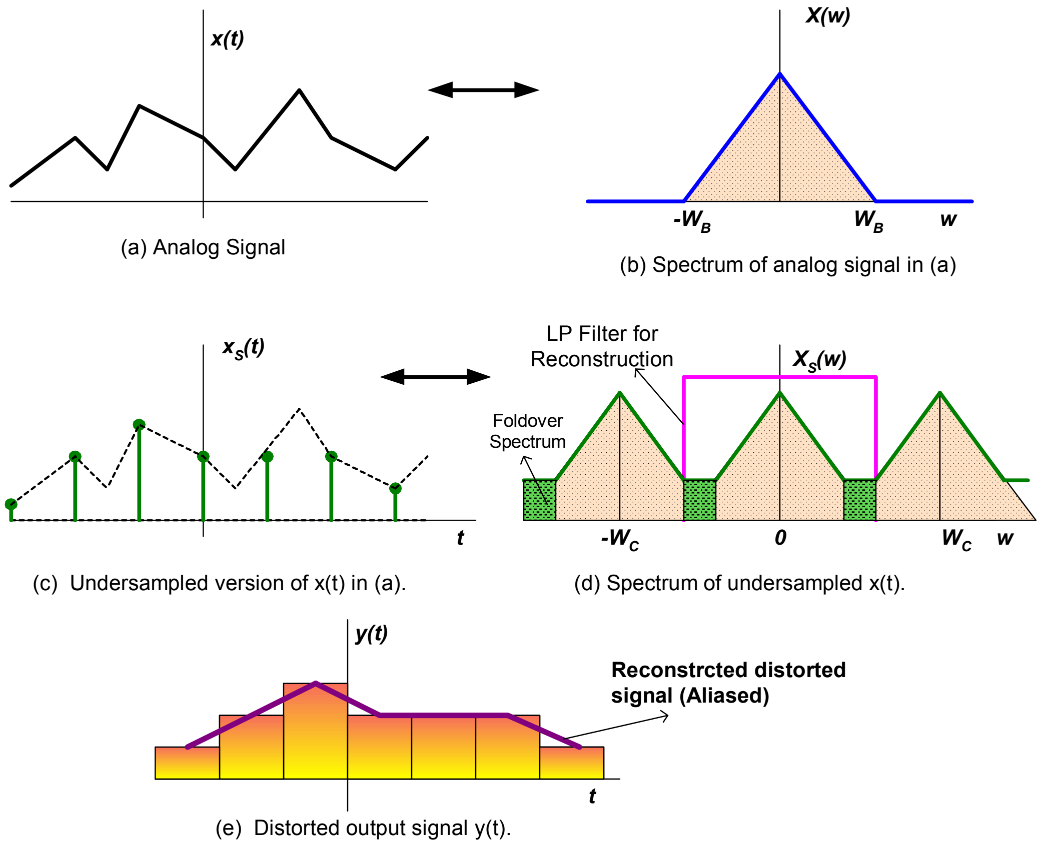


Figure 6.10. Effects of under-sampling (aliased) on reconstructed signal.

In order to use ideal sampling (perfect interpolation) without any spectral fold (aliasing distortion), one needs to have a signal which has to be:

- Time-limited (needed for transmitting at a information rate) and
- Bandlimited (needed for perfect reconstruction).

Example 6.3 PAM Sampling with flat-top pulses: Let us use a pair of perfect switches to act as a natural sampler, which is often the case in many communication systems:

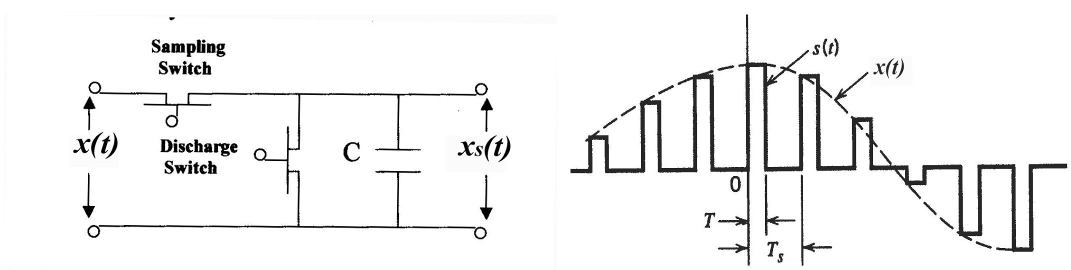


Figure 6.11 Practical flat-top PAM sampling using two-transistor switches.

This is known as the Pulse Amplitude Modulation (PAM) in the business and it is easier to generate in practice by using an analog switch and a clock circuit, which are readily available in CMOS and other VLSI circuitry. This class of PAM with natural sampling is used mostly in fast conversion tasks, which do not have a digitization stage to follow. The analog waveform is recovered by using a mixer followed by a low-pass filter as in the ideal case.

In this set-up, the PAM signal is multiplied with a sinusoidal oscillator output to shift the spectrum of the modulated band-pass signal to baseband. In addition to the baseband component, the mixer generates higher order harmonics of the spectrum and a properly designed low-pass filter suppresses all harmonics but the baseband to generate the replica of the original waveform. The second task of this filter is to suppress the spurious frequency components due to non-linearities in the PAM generation and recovery circuits.

In addition to PAM, there are three other forms of pulse modulation. Two of these: Pulse Width Modulation (PWM) and Pulse Position Modulation (PPM) are shown in Figure 6.12 together with the corresponding PAM flat-top sampling. The third technique is the most popular Pulse Code Modulation (PCM) and it will be discussed in detail since it suits perfectly to the Time Division Multiplexing (TDM) framework and hence, forms the basis for modern digital communication systems and telecommunication services.

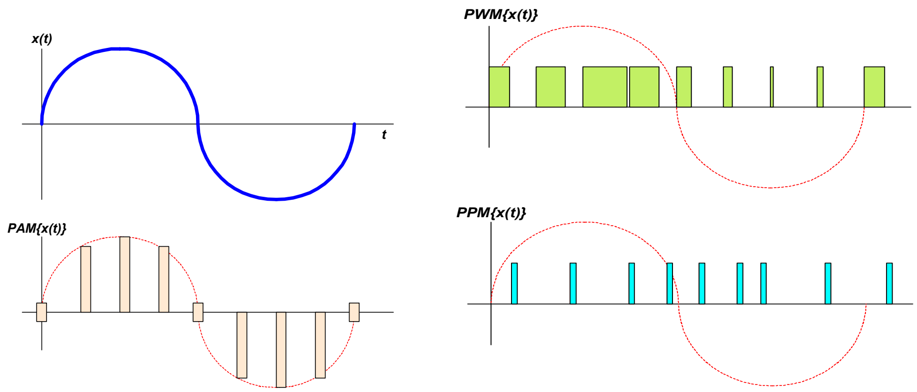


Figure 6.12 PAM. PWM and PPM sampling of analog signals.

6.2 Time Division Multiplexing (TDM) Infrastructure

An important feature of all of these pulse modulation schemes is the fact that the pulse carrying information is "*ON*" only a fraction of the sampling period and it is zero rest of the time. If we clear these zero intervals to place pulses from other communication systems, then we can transmit information from multiple sources over the same channel. In other words, we perform data fusion from a number of sources into a one big data sequence. This is commonly known as the Time-Division Multiplexing (TDM) in the industry. Actually, we can do the data fusion in the frequency-domain by taking transforms of the signals from sources to be bundled together. This, in turn, results in the frequency-division multiplexing (FDM). Along the same line, we can do Code-Division Multiplexing (CDMA), space multiplexing (SDMA), and finally, wavelength multiplexing (WDMA) to use the channel more efficiently.

Among these, TDM is the most frequently used configuration since it forms the basis for the national and international PCM infrastructure. For instance, 24 voice channels are multiplexed together to form the T-1 layer of the North American and Japanese telecommunication networks. But, 32 are similarly multiplexed to form E-1 level in the ITT system used throughout the world.

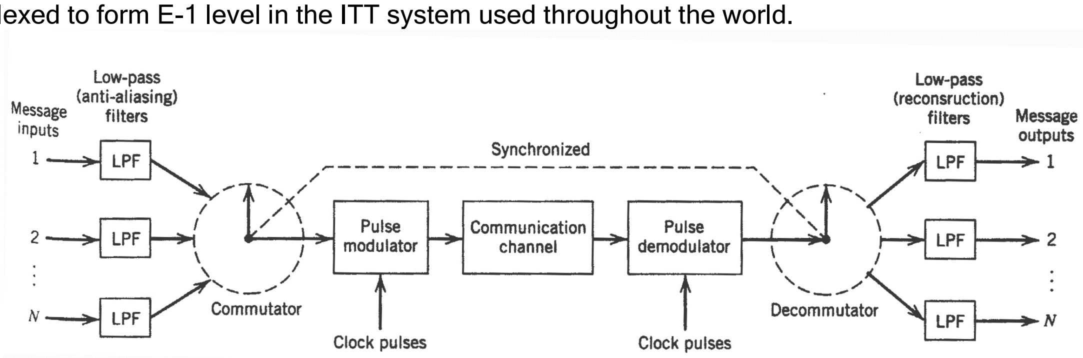


Figure 6.13a Illustration of Time-Division Multiplexing (TDM).

In Figure 6.13a, we illustrate the notion of TMD in terms of commutators and decommutators. Here, signals from N message channels are sampled properly at their Nyquist rate or above. As shown in Figure 6.14b, suppose that the pulse widths of these samples are very small with respect to their period. Then we can employ a commutator to place pulses from these two signals alternately in tandem. If the synchronization information for the commutator, i.e., the precise manner the commutator works, and the way the binary representations of samples are stung together - is available to the decommutator at the other end then we can target the data to their appropriate users (Figure 6.14). In order to generate parallel data out of this series pulses we need a decommutator at the receiver working in sync with the transmitter side.

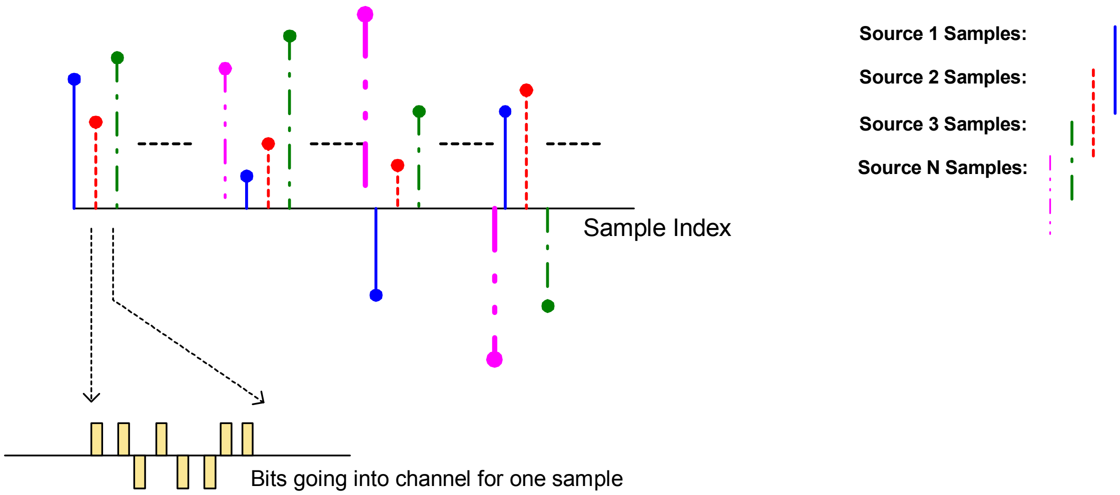


Figure 6.13b Fusion of quantized data from N sources into a TDM data string using Commutator.

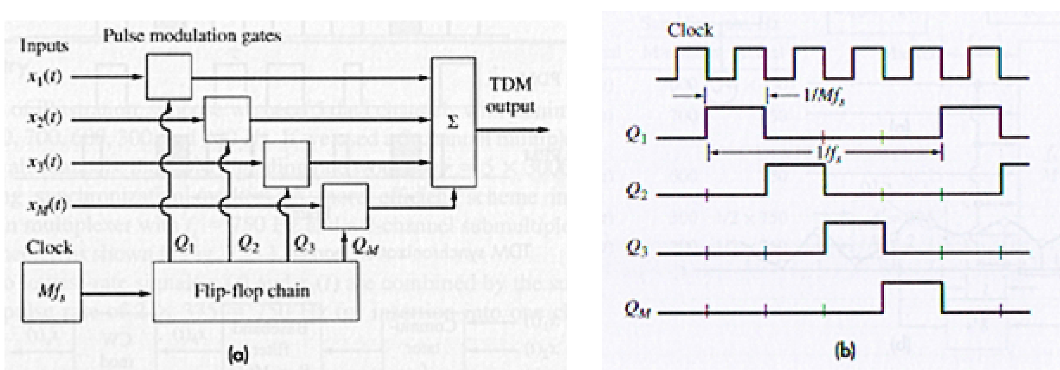


Figure 6.14 TDM Commutator (a) and timing diagram (b).

The minimum bandwidth of a TMD baseband signal is very easy to determine from the Nyquist theorem conditions. Suppose that the TDM data sequence contains $2B_i$ samples per second from the i^{th} channel from a total of N channels and B_i is the corresponding bandwidth for that channel. The total number of samples in the sequence is the arithmetic sum of data:

$$n_S = \sum_{i=1}^{N} 2B_i \qquad (6.13)$$

If the channels have the same bandwidth B Hz, then the overall bandwidth for an N-channel TDM baseband system will be:

$$B_T = 2NB \quad Hertz. \qquad (6.14)$$

6.3 Pulse Code Modulation (PCM) Infrastructure

PCM systems constitute the backbone of the existing public telecommunication hierarchy throughout the world. There are basically two basic infrastructures: the North American and Japanese networks based on an aggregate transmission rates at integer multiples of 1.544 MB/s over T-1 lines and the CCITT networks based on integer multiples of 2.048 MB/s E-1 lines. This configuration is commonly known as the "*plain old telephone service (POTS*" in the telecommunication industry.

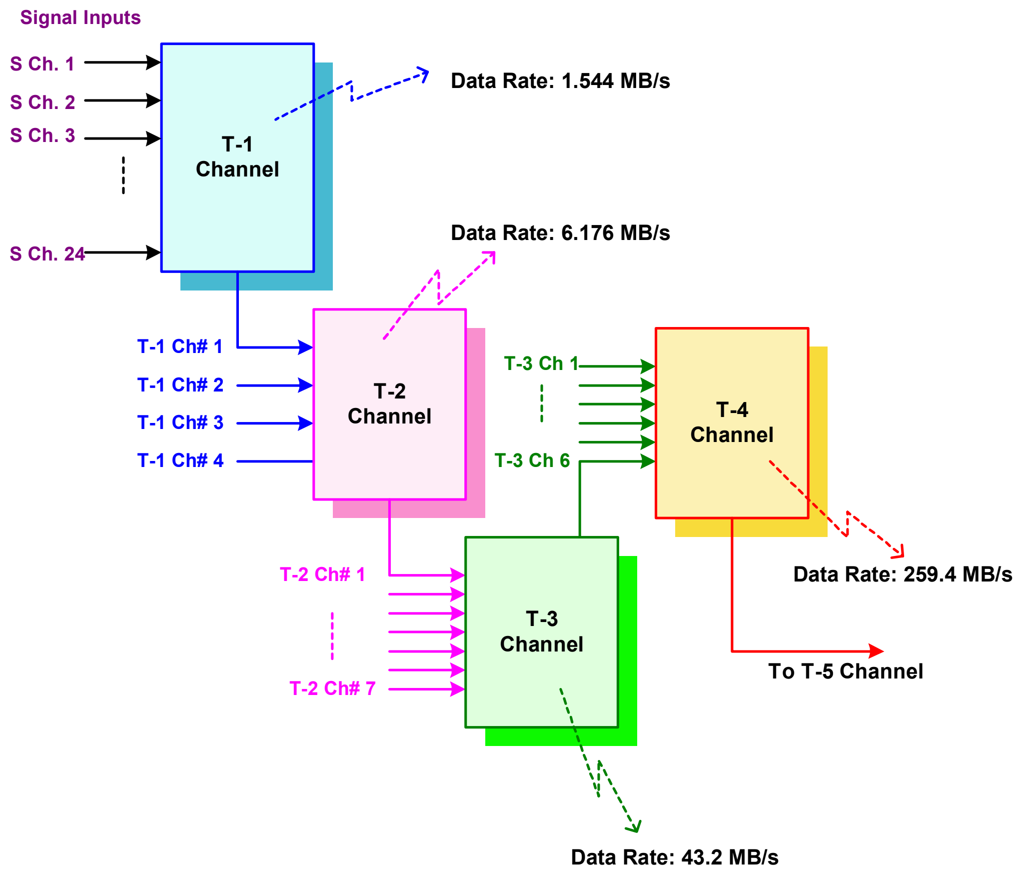


Figure 6.15 North American and Japanese TDM hierarchy for digital plain old telephone service (POTS).

The first stage in this architecture is the T-1 Carrier System, where a set of 24 voice-grade signals sampled at 8,000 samples per second and a resolution of 8-bits per sample corresponding to a data rate of 64,000 b/s is time-multiplexed. The overall bandwidth and the associated bit rate is 1.544 MB/s, where 1.536 MB/s is for data from 24 voice channels and the remaining 8.0 kb/s is for framing and synchronization. T-1 information is transmitted over nominally 22 - 25 gauge copper wire pair and it is mostly used in the terrestrial networks.

The international networks, however, has a similar infrastructure where the building block is the 32-Channel E-1 Carrier system. Here 30 voice grade channels are TDM multiplexed together with two control, protocol, and synchronization channels. Each with a bit rate of 64 kb/s results in an overall bit rate of 2.048 MB/s and the bandwidth requirement is appropriately increased to 2.048 MHz.

At present, the TDM systems have been generally replaced by Time-Division Multiple Access (TDMA), especially, in communication satellites and cellular communication systems. Since not every channel is always "on-line" the channel capabilities are fully utilized. In order to keep the overall system almost always "FULL," neat procedures are developed to accept data from higher than 24 (or 30 in CCITT systems) according to some statistics-based switching operations. This is done at the expense of adding a buffer to the system. Buffer size plays a key role since a large buffer could be unacceptably costly, whereas, a very small one can easily overflow. Independent of the size, when a demand is not met due to overflow, it is not a good business practice in these days of stiff competition and dropping service charges.

6.3.1 End-to-End Single Channel PCM System:

If we take only one of these voice grade input signals and go through the complete process of communicating over a PCM system, this is called an end-to-end single channel PCM configuration in the engineering jargon. A general block diagram for a voice grade speech communication over telephone lines is shown in Figure 6.16. Here we see that the first step is to bandlimit the analog signal with a low-pass filter to have an effective bandwidth of B=4,000 Hz. This filter is an analog anti-aliasing filter with a cut-off frequency of 3300 Hz and the speech is suppressed to a minimum of 35 dB at 4.0 kHz. The next step is to obtain samples of this baseband signal at 8,000 samples per second using a flattop PAM sampling as discussed earlier.

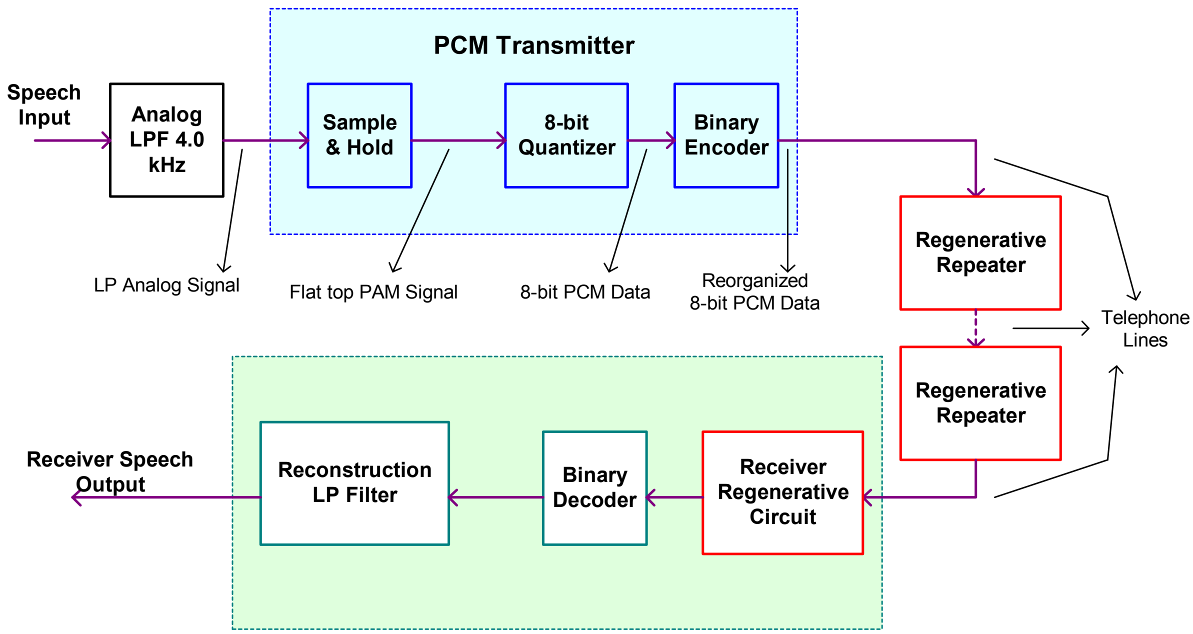


Figure 6.16 End-to-end single channel PCM block diagram.

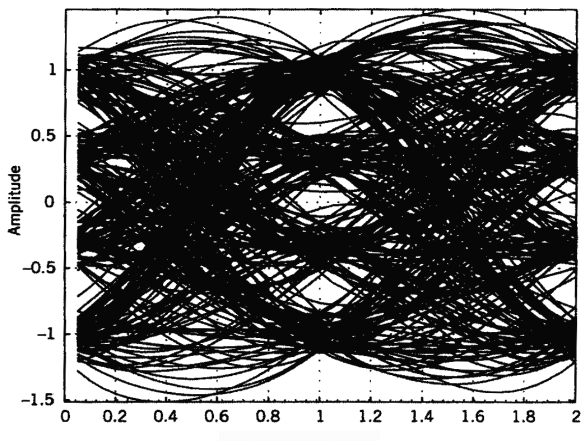
Let us assume that the quantizer has *M*-levels ($M=2^m$) to permit the amplitude variations to be encoded by *m*-bits long codewords. The output of the quantizer is sometimes encoded by another encoder to built codes resilient to channel errors. One such code is a Gray code, where the adjacent codewords differ only in one location as shown in Table 6.1 for *m*=3 or 8-level binary quantizer.

Quantized Sample Amplitude	Binary-level Representation	Gray Code
+7.0	111	110
+5.0	110	111
+3.0	101	101
+1.0	100	100
-1.0	011	000
-3.0	010	001
-5.0	001	011
-7.0	000	010

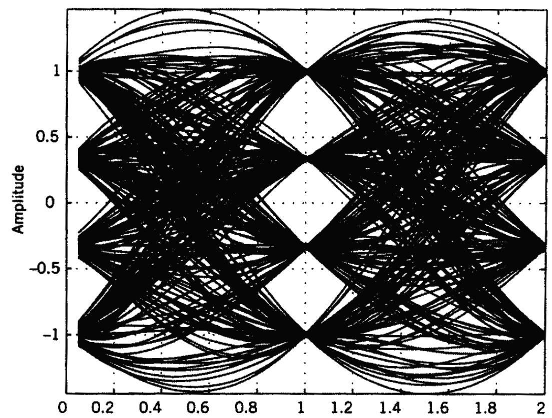
Table 6.1 Three-bit (8-level quantization levels and Gray (Manchester) Code

The key feature of Gray (Manchester) code is the fact the adjacent codewords has only one bit difference. The output of the encoder is scrambled and encrypted in the case of secure communications. In the case of public telephone services, however, it is multiplexed with 23 or 29 other PCM encoding systems. The resultant bit streams are transmitted over the allocated transmission medium using an adopted pulse shape. In the case of terrestrial networks, PCM signals need to be detected synchronized and reissued every 1800 meters or so using “regenerative repeaters.” In the case of microwave and satellite communications, however, there are radiolink relay stations and the relay functions in the existing geo-stationary telecommunication satellites to achieve that. The operation of the receiver is just the opposite of the transmitter side.

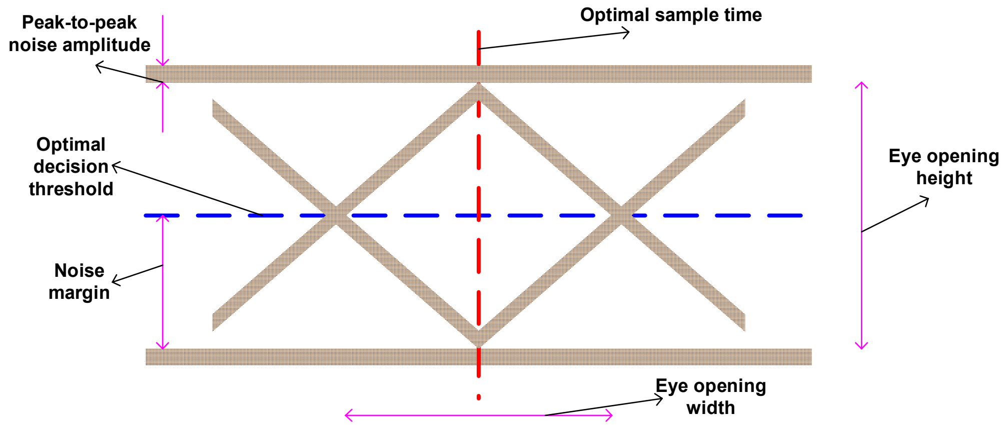
However, if the receiver clock frequency or the timing is not perfectly in synchronization with that of the transmitter, of if the signal is dispersed in the channel then the exact recovery becomes an issue. This is due to the fact that while the sinc pulse used for synthesizing the current sample is peaking the pulses from neighboring samples do not have zero-crossings at that moment.



a. Eye Diagram for a corrupt channel



b. Eye Diagram for a perfect channel



c. Eye Diagram Terminology

Figure 6.17 Open and closed eye diagrams and the terminology

We can clearly see in Figure 6.17a that a signal has been subject to noise and other degradations, such as intersymbol interference, cross-talk, echoes, etc., as it traverse towards receiver. However, if we have a perfect sync and no noise in the channel then the eye diagram would be an ideal case as in Figure 6.17b. The peaks and zero-crossings are clearly observable. This is called an open eye. We illustrate the terminology associated with eye diagrams in Figure 6.17c.

To elaborate further let us consider the plot of a 6-bit long data sequence and the signal for this string as shown in Figure 6.18a. Their decision boundaries are marked as well as the decision moments. It is more illustrative to take the signal from each symbol interval and align them with respect to their sampling moments as it is normally done on an oscilloscope. This is shown in Figure 6.18b. With this we have just created an eye diagram for these six bits. As we clearly see from this figure the eye is no longer perfect. If the communication regime is very noisy; and/or the rate of intersymbol interference, and/or cross-talk from other channels, and/or the echoes are severe then we get the eye closed as mentioned earlier while discussing Figure 6.17a.

The combat the ills in the channel or improve the eye-openings we attack from two fronts:

- to design better pulses by replacing rectangular pulses with pulse shapes which are more robust and
- to use equalizer filters at the receiver front end.

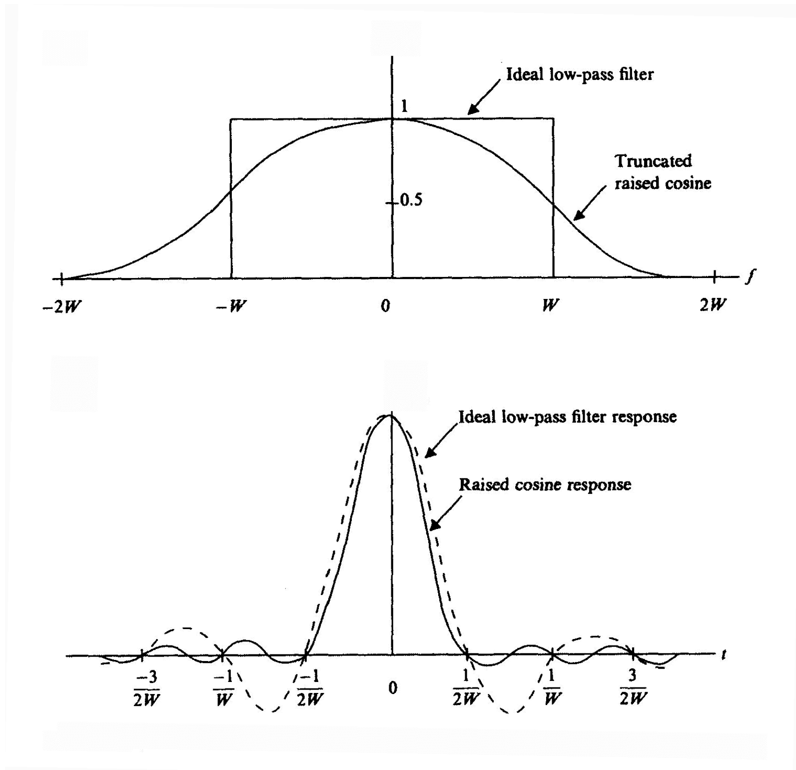


Figure 6.18 Frequency and impulse responses of a raised cosine pulse.

One such pulse shape is the industry standard raised cosine pulse and its frequency and impulse responses are shown in Figure 6.18. By stretching the bandwidth from W to $W+\alpha$ the impulse response rolls-off faster and the net effect is the signal during the reconstruction is more pronounced. The bandwidth expansion α is normally between 25-50% of the original bandwidth. The equalizer concept will be discussed in later chapters. The quality of the PCM system depends on the number of bits in the quantizer codewords. Higher the encoding rate (m) smaller the difference between the original signal and the reconstructed signal from samples as shown in Figure 6.3 earlier.

6.3.2 Uniform Quantizer: he most critical block in all digital communication systems is the quantizer, where discrete samples (flat-top PAM signal) of an analog waveform is amplitude digitized into m-bits long codewords. Quantizers are generally into uniform and non-uniform coders. In the first case, all step sizes in the digitization step are equal as it was depicted in Figure 1.6. Let us visit the issue in Figure 6.19.

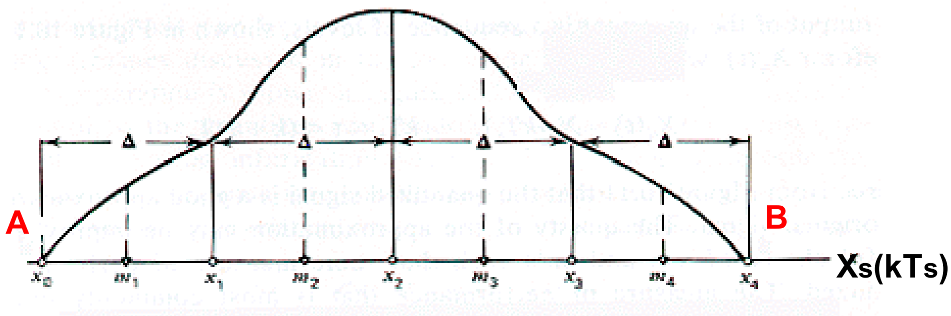


Figure 6.19 m-Bit uniform quantizer

In this case, let us assume that A and B are the minimum and maximum values of $X_S(kT_S)$, respectively, and then the uniform quantizer will have a step size:

$$\Delta = (B - A)/L \qquad (6.15)$$

Here L is the total number of steps and is related to the resolution of the quantizer by

$$L = 2^m \qquad (6.16)$$

where m is the resolution in number of bits. If the value of the signal at sampling instant $t = T_S$, $X_S(kT_S)$, falls in the i^{th} quantizing interval, the quantizer output values will be the midpoint of that particular interval. That is,

$$X_q = m_i \quad \text{if} \quad x_{i-1} < X \le x_i \qquad (6.17)$$

where

$$x_i = A + i.\Delta \quad \text{and} \quad m_i = \frac{x_{i-1} + x_i}{2}, \quad i = 1,2,\cdots,L \qquad (6.18)$$

These expressions together are known as the encoding process and there is an inverse process for the decoder as expected. In Figure 6.20, we present a 4-bit (16-level) uniform quantizer circuit using OPAMPs, FlipFlops, resistors as multiplicative factors, an AND-gate and a clock. In Figure 6.21, the quantizer input and the encoded output codewords from this circuit is plotted. Finally, we elaborate numerically the quantization process of Figure 6.21 in Table 6.2.

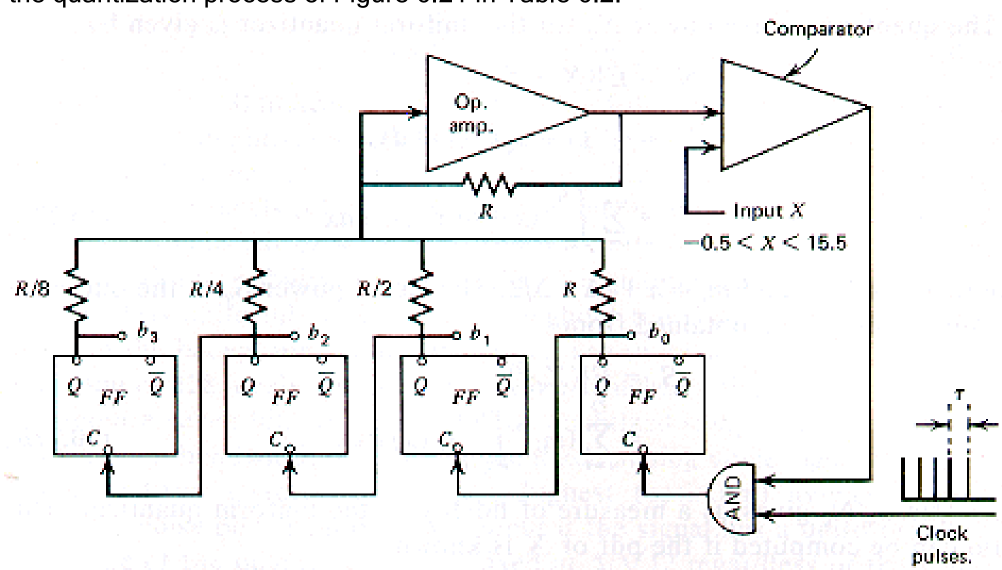


Figure 6.20 4-Bit (16-level) uniform quantizer circuit using discrete components.

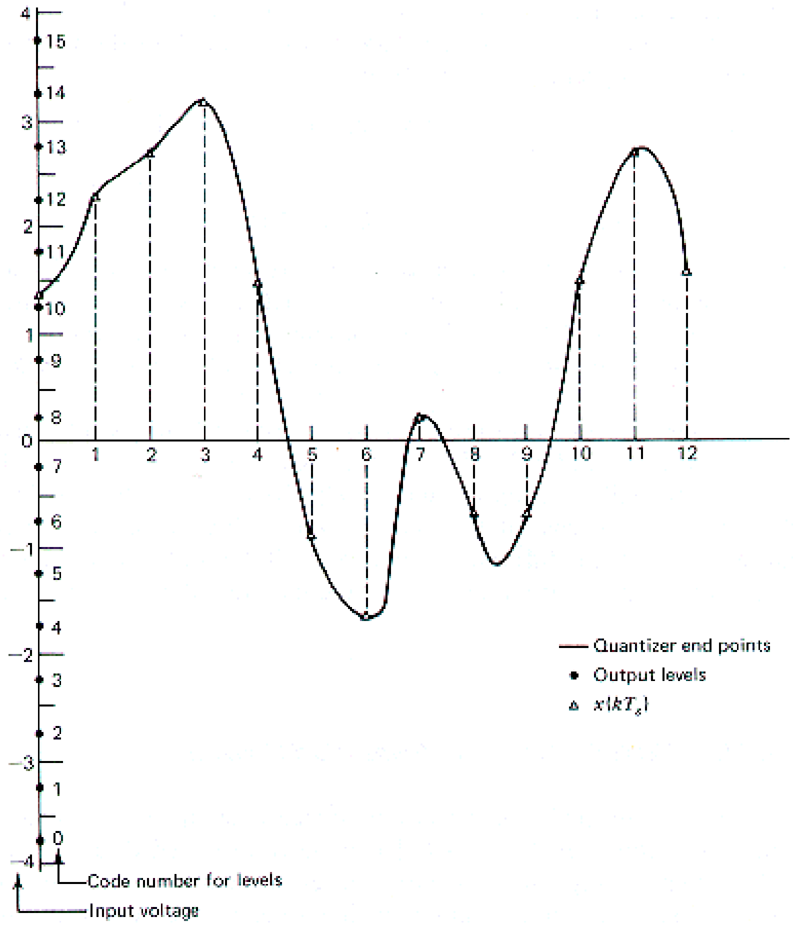


Figure 6.21 Quantization of an analog signal x(t) with the 4-Bit uniform quantizer of Figure 6.20.

Table 6.2 Quantization Values for 4-bit Uniform Quantizer of Figure 6.21

Sampled values of an analog signal	1.3	2.3	2.7	3.2	1.1	−1.2	−1.6	0.1	−1.2
Nearest quantizer level	1.25	2.25	2.75	3.25	1.25	−1.25	−1.75	0.25	−1.25
Level number	10	12	13	14	10	5	4	8	5
Binary code	1010	1100	1101	1110	1010	0101	0100	1000	0101

6.3.3 PCM Receiver with a Uniform Decoder: In the receiver side we need to have a decoding and reconstruction process to undo all the processing done at the transmitter. In Figure 6.22, we present the major components of the receiver system. To combat the ills of the channel and to improve the eye-opening for better recovery, an equalizer filter (matched-filter receiver) is used as the first block, which attempts to make a simple decision of “0” or “1” from the incoming signal. The remainder of the circuitry should be self-explanatory.

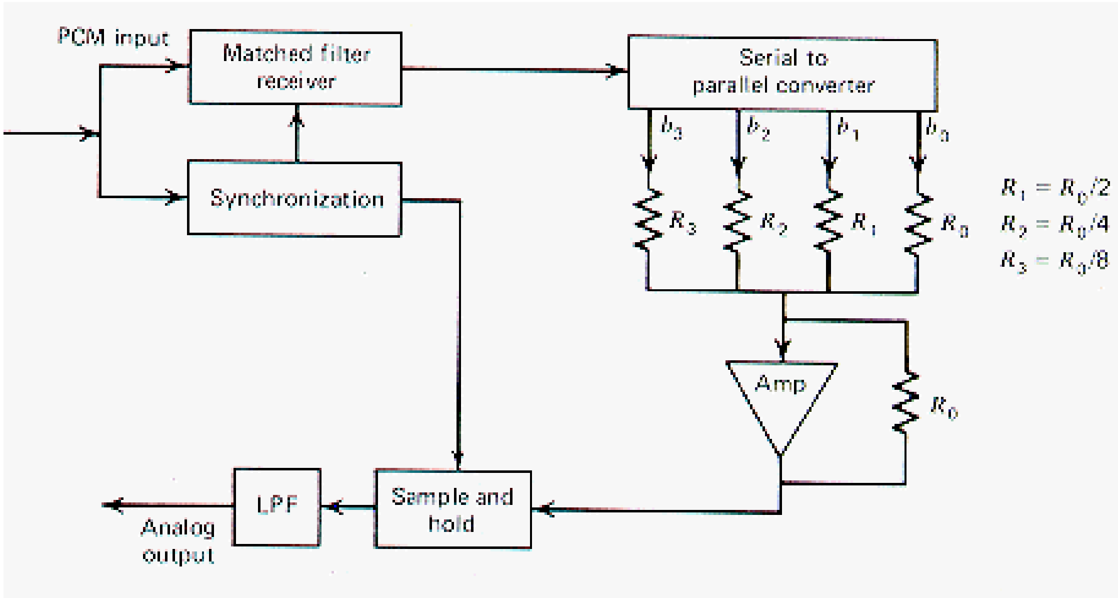


Figure 6.22 Components of a PCM Receiver for a PCM System with Uniform Quantizer.

6.4 Performance of a PCM System

As in all digitization schemes there is error in using quantized values and that is not recoverable. This is called "Quantizing Distortion (Noise)" and it is the dominant issue in the source compression task of the overall communications link. In addition, there will be interference from neighboring channels, and friendly and unfriendly jammers. Finally, noise in the transmission medium will be affecting the quality of reception of the signal. The first degradation, quantizing distortion, is a source-coding problem and whatever happens in the channel has no bearing on that. In the next few paragraphs, we will be studying the effects of the last two disturbances and their remedies.

6.4.1 SNR for PCM Systems: Let $x(kT_S)$ and $\hat{x}(kT_S)$ be the actual k^{th} sample and its quantized version, respectively. If we were to reconstruct the analog replicas of these we would use the *sinc* function reconstruction formulas as previously discussed.

$$x(t) = \sum_{\infty} x(kT_S).Sinc(2Bt - k) \quad (6.19a)$$

$$\hat{x}(t) = \sum_{\infty} \hat{x}(kT_S).Sinc(2Bt - k) \quad (6.19b)$$

The quantizing distortion (noise) will be simply the difference between these two signals:

$$\begin{aligned} q(t) \equiv \hat{x}(t) - x(t) &= \sum_{\infty} [\hat{x}(kT_S) - x(kT_S)].Sinc(2Bt - k) \\ &= \sum_{\infty} q(kT_S).Sinc(2Bt - k) \end{aligned} \quad (6.20)$$

where $q(kT_S)$ is the quantizing error in the k^{th} sample.

The quantizing noise error waveform itself is not very useful since it is a time-dependent quantity. However, its statistics are very important in communications. The first-order statistic is the mean or the ensemble average and it is usually assumed zero. Even if it is not, then there are very simple mean-removing techniques to force it to zero. That is:

$$m = \mu_m = \overline{q(t)} = 0 \quad (6.21)$$

The second-order statistic is the mean-square (MSE) value of the noise, which is directly related to the quantizing noise power. If *T* is the overall observation time then:

$$\overline{q^2(t)} = Lim_{T\to\infty}[\frac{1}{T}.\int_{-T/2}^{T/2} q^2(t)dt = Lim_{T\to\infty}[\frac{1}{T}.\int_{-T/2}^{T/2} [\sum_{\infty} q(kT_S).Sinc(2Bt-k)]^2 dt \qquad (6.22)$$

However, if we interchange the order of integration and summation and use the following result from definite integration tables:

$$\int_{-\infty}^{\infty} Sinc(2Bt-m).Sinc(2Bt-n)dt = \begin{cases} 1/2B & if\ m=n \\ 0 & if\ m\neq n \end{cases} \qquad (6.23)$$

We obtain the expression for the noise power:

$$\overline{q^2(t)} = Lim_{T\to\infty}[\frac{1}{2BT}.\sum_{\infty} q^2(kT_S)] \qquad (6.24)$$

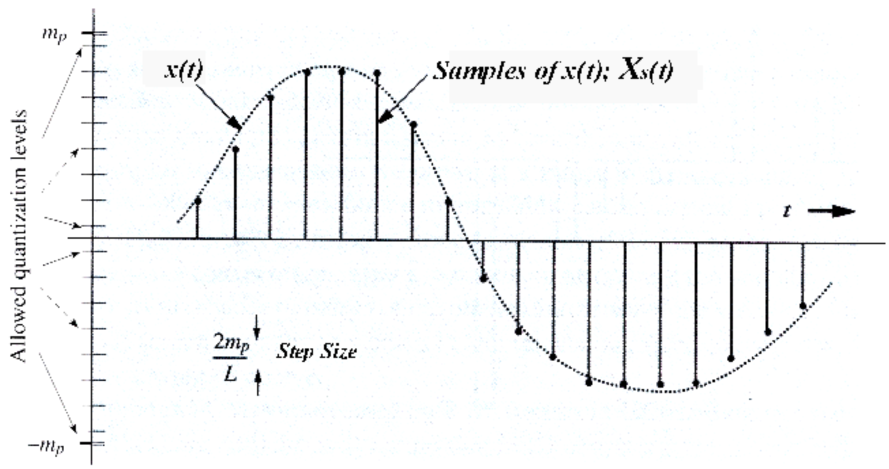


Figure 6.23 PCM using an *M-level* uniform quantizer.

Each sample is represented by the mid-point of each step as it was discussed earlier and depicted in Figure 6.23 (Note that this is the same picture discussed in Figure 1.6) $\Delta v \equiv 2m_P/L$ in the quantizer, where m_P is the peak value of the quantized signal and L is the number of levels in the quantizer. In this case, the maximum quantization error for each sample lies in the range: $(-\Delta v, \Delta v)$. If we assume this error is uniformly distributed (equally likely) in this interval then the mean-square (MSE) value of the error can be written as:

$$\overline{q^2(t)} = \frac{1}{\Delta v}.\int_{-\Delta v/2}^{\Delta v/2} q^2(t)dt = \frac{1}{\Delta v}.\frac{q^3(t)}{3}\Bigg|_{-\Delta v/2}^{\Delta v/2} = \frac{(\Delta v)^2}{12} = \frac{m_P^2}{3.L^2} \qquad (6.25)$$

This last result is called the "Uniform Quantizer Noise Power:"

$$N_q \equiv \overline{q^2(t)} = \frac{(\Delta v)^2}{12} = \frac{m_P^2}{3.L^2} \qquad (6.26)$$

Similarly, we write the "Signal Power" as the mean-square value of the zero-mean input signal:

$$S_0 \equiv \overline{x^2(t)} \qquad (6.27)$$

The Signal-to-Distortion-Ratio (SDR) or commonly known as the signal-to-noise ratio (SNR) is defined by:

$$SDR = SNR \equiv \frac{S_0}{N_q} = 3.L^2.\frac{\overline{x^2(t)}}{m_P^2} \qquad (6.28a)$$

or more commonly known form in terms of decibels:

$$SDR_{dB} = SNR_{dB} \equiv 10.0 * Log_{10}(\frac{S_0}{N_q}) \qquad (6.28b)$$

These last two formulas are not in a form the engineering community can easily compare with the performance of analog systems. However, for a 4σ loading condition, that is, when four standard deviations of $x(t)$ is used as m_P results in a simple approximation of this formula:

$$SDR_{dB} = SNR_{dB} = (\frac{S_0}{N_q})_{dB} \approx 6.n + \alpha \qquad where \quad \alpha = 10.Log_{10}(3.\frac{\overline{x^2(t)}}{m_P^2}) \tag{6.29}$$

The parameter α varies from talker-to-talker up to 40-dB (10^4). It also varies due to different circuit lengths of the communication links between the source and the user. Finally, SNR can vary due to changes in the speaking characteristics from time-to-time.

6.4.2 Transmission Bandwidth of PCM Systems:

Let the signal $x(t)$ have a bandwidth B Hz at the output of the anti-aliasing filter, a sampling rate of 2B samples per second and each sample is encoded into $m = Log_2(L)$ bits, where L is the number of levels in the quantizer. The transmission rate for this setup will be:

$$Rate = R = 2mB \text{ Pulses/s} \tag{6.30}$$

Using a bandwidth factor in the range $1 < k \le 2$ for the channel will require kmB Hz bandwidth in PCM systems. Let us now elaborate that wit a specific example.

Example 6.4: Given a PCM system with 256-levels and bandwidth of 4.0 kHz, find the channel bandwidth needed to transmit this signal.

$$L = 256 \quad \Rightarrow \quad m = 8 \; bits/sample$$

$$B = 4.0 \; kHz \quad \Rightarrow \quad F_S = 8{,}000 \; samples/s$$

$$Bandwidth \; per \; Channel = 64 \; kHz$$

$$24 - channel \; T - 1 \; link \; bandwidth = 64{,}000 * 24 = 1.536 \, MHz$$

In addition, 8000 Hz is added for framing and signaling (synchronization) to bring the total bandwidth to:

$$Transmission \; Bandwidth = 1.544 \; MHz.$$

Example 6.5: Compare the above bandwidth that with that of a single CD-audio recording. In CD industry, audio is band-limited to 11,025 Hz and sampled at twice the Nyquist rate and coded into one of $L = 2^{16} = 65{,}536$ levels. Find the information rate and the necessary bandwidth for one second of recording.

$$F_S = 4.(11{,}025) = 44{,}100 \; Hz$$

$$Rate = Log_2(L).F_S = 705.6 \; kbp$$

and the bandwidth range is: $705.6 \; kHz \le W \le 1.411 \; MHz.$

6.4.3 Bandwidth-SDR Tradeoff for PCM Systems: An analysis of (6.23) and (6.24) show that a 6.0-dB increase in SDR is achieved by going from n-bits per codeword to $(n+1)$-bits. The impact of this on the bandwidth is an increase of 8,000 Hz. Thus, PCM system performance can be improved by 6.0 dB at the expense of an increase of 8.0 kHz bandwidth per channel. This is known as the SNR/Bandwidth Tradeoff in the engineering community.

6.5 Non-Uniform Quantizers

The uniform quantizer used in the previous section has exactly the same step-size for each level and it is used in applications where data compression is not the primary motive. In most public and secure communication systems, we deal with signals, which exhibit non-uniform amplitude distribution. Particularly, smaller amplitudes dominate the transmitted sample sequence in traditional communication, whereas large amplitudes are very rare. In other words, quantizer steps at the center of the staircase are heavily used and the tail levels are very infrequent. To save bandwidth one idea is to truncate these large values since they are very infrequent. However, we pay the price of loosing details in video or richness in sound. Instead, we attempt to exploit this infrequency for the purposes of improved SDR or reduced bit rate by assigning non-uniform step sizes. This is due to the fact that there will be more steps in the center and the range of the error signal will be smaller. This is demonstrated in Figure 6.24.

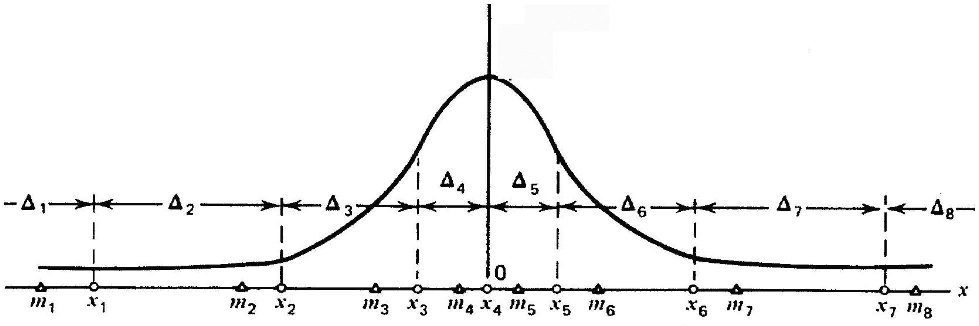


Figure 6.24 Illustration of a 3-bit non-uniform quantization.

Here we have an 8-level quantizer with a codeword set $\{m_1, m_2, \cdots, m_8\}$ placed unevenly along the horizontal axis. Smaller *x* values are mapped more precisely to their corresponding codewords than the larger input levels, which can be seen from the step-size set $\{\Delta_1, \Delta_2, \cdots, \Delta_8\}$. The unevenness has to be carefully developed according to some underlying principle. One possible technique would be to study the statistics of the signals to be quantized and distribute the levels according to the statistics. There are three well known classes of quantizers use this approach:

(1) Lloyd-Max non-linear quantizers, where each step-size optimized according to the underlying statistical distribution. For instance, an exponential distribution is assumed for the intensity levels of pixels in an image frame.
(2) Generalized Lloyd I type quantizers, where the levels are matched to the statistics of large training databases. These quantizers are usually multi-dimensional and they form the basis for Vector Quantization (VQ) in modern communication systems.
(3) Obtain optimal quantizer levels is to pass the signal through a companding network, whose output is a uniformly distributed signal as in Figure 6.25.

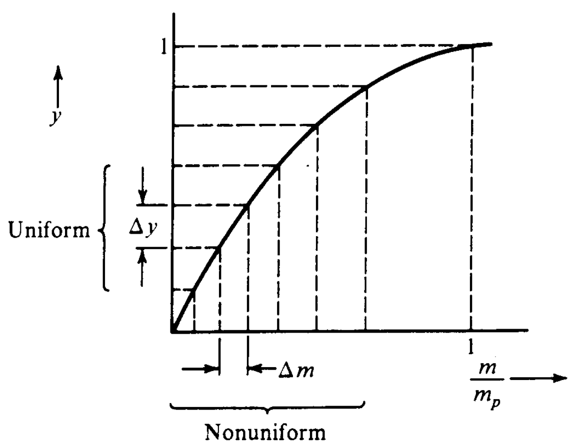


Figure 6.25 A Compandor for mapping non-uniform input levels to uniform ones.
(Reprint from Lathi's text, courtesy of Oxford Press)

As it is clear from above, the input signal falling into regions with non-uniform lengths Δm, which are increasing as the amplitude increases, are mapped into uniform regions with range Δy. The system, which does this type of transformation, is called a *compandor*. It should be expected that the precisely the opposite procedure will need to be performed in the receiver by an *expandor*.

This approach has been the norm in the existing telecommunication infrastructure as a result of many studies performed by the phone companies in the period 1960-1975. According to these long-term studies on telephone speech samples, it observed that speech samples exhibit roughly an exponential distribution, where samples with small amplitude occur exponentially more often than the larger ones. To exploit this exponential character, input speech is processed by a logarithmic network and its outputs are expected to be significantly more uniform. Next, we pass these logarithmically compressed signals by a uniform quantizer of the previous section. In the receiver, however, samples are expanded by an exponential network to cancel out the compression. This process is called *Companding*. There are two logarithmic laws used for companding voice/audio grade signals, namely, the *A-Law* for the international circuits and the $\mu - Law$ for the North American and Japanese systems. The input-output characteristics of these two laws are shown below.

Case 1: CCITT Law with A=87.6: If the input signal is $x(t)$ with a peak amplitude level m_P the output signal is given by:

$$y(t) = \begin{cases} (\frac{A}{1+\log_e A})\frac{x(t)}{m_P} & if \quad |x(t)/m_P| \le 1/A \\ Sgn(x(t)).\{\frac{1+\log_e(A.|x(t)/m_P|}{1+\log_e A}\} & if \quad 1/A \le |x(t)/m_P| \le 1 \end{cases} \tag{6.27}$$

Case 2: $\mu - Law$ **with** $\mu = 255$:

$$y(t) = Sgn(x(t)).\frac{log_e[1+\mu.|x(t)/m_P|]}{log_e(1+\mu)} \qquad if \quad |x(t)/m_P| \le 1 \tag{6.28}$$

In either case, the signal-to-quantizing distortion ratio: S_0/N_q is nearly constant over most voice-grade input signal with a power range of 40 dB. For instance, the output SDR for the $\mu - Law$ can be approximated by:

$$\frac{S_0}{N_q} \approx \frac{3L^2}{[\log_e(1+\mu)]^2} \qquad if \quad \mu^2 > \frac{m_P^2}{\overline{x^2(t)}}$$

where L is the number of quantizer levels and $\mu = 255$ is uniformly used in practice. Similar to non-compressed case, this result can be rewritten by:

$$\frac{S_0}{N_q} \approx 3k.2^{2n} \qquad where \quad k = \frac{1}{[\log_e(1+\mu)]^2}$$

or equivalently,

$$SNR_{dB} = (\frac{S_0}{N_q})_{|dB} \approx 6n + \alpha \qquad where \quad \alpha = 10\log_{10}(3k) \tag{4.29}$$

The plot of this curve for $\mu = 255$ together with that of a PCM system without companding, i.e., $\mu = 0$ is shown in Figure 6.26.

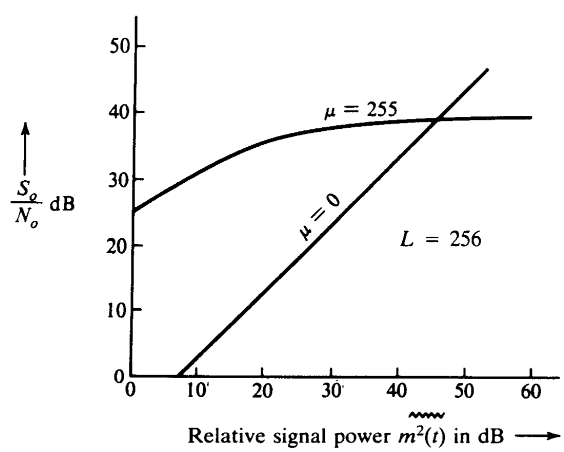


Figure 6.26 Performance of a companded and uniform quantizers.
(Reprint from Lathi's text, courtesy of Oxford Press)

It is clear from these plots that companded systems outperform the uniform case significantly for small SDR cases, which is the predominant situation observed in most real-life applications. This improvement is over 12 dB for signal powers of 20 dB or less

Example 6.6: Consider the PCM system with 64-levels $\mu = 100$ quantizer and bandwidth of 4.0 kHz.

Per channel bandwidth: 2nB = 48.0 kHz.

The system SDR is expected to be:

$$SDR \approx \alpha + 6 * 6.0 = \alpha + 36$$

$$\alpha = 10 \log_{10} \frac{3}{[\log_e(101)]^2} = -10.1\, dB$$

$$SDR \approx 25.9\, dB\ .$$

If we increase the resolution to n=8, 12, 16 bits, respectively, the performance would jump to:

$$SDR \approx 37.9,\ 61.9,\ and\ 85.9\ dB,\ respectively.$$

Tradeoff on increased bandwidth would be:

$\Delta BW = 16\ kHz,\ 48\ kHz,\ and\ 80\ kHz$, respectively.

Example 6.7: $\mu - Law$ PCM Quantizer implementation in Matlab. Quantize a sequence of 500 Gaussian distributed random numbers generated with $N(0,1)$, where mean=0 and variance=1, using a $\mu - Law$ PCM Quantizer with 16, 64, 256 levels. Plot the error and obtain the SQNR in each case. The m-file as well as the plots generated for this example is included at the end of the chapter. The signal-to-quantizing noise ratios are, $SQNR_{16}$ = 12.37 dB, $SQNR_{64}$= 25.6 dB; $SQNR_{256}$= 37.5 dB, respectively.

6.6 Differential Pulse Code Modulation (DPCM)

In PCM communication systems every sample is encoded regardless of its past and future. In other words, PCM is a memoryless coding scheme where the coder does not remember anything from the past of the signal being processed. On the other hand, signals in nature have varying degree of redundancy. Signal redundancy is normally measured in terms of correlation among neighboring samples as shown in Figure 6.27. For instance, speech samples are known to exhibit 80-90 percent correlation between adjacent samples. This correlation and the inherent redundancy can be exploited for achieving lower transmission data rate or better performance at a pre-specified transmission rate.

One of the earliest notions in the field of data compression has been to code the difference between two adjacent samples rather than the sample amplitudes themselves. These systems are called *Differential Coding* schemes. Differential Pulse Code Modulation (DPCM) and Delta Modulation (DM) and their variations are the best-known traditional differential coders for compressing waveforms.

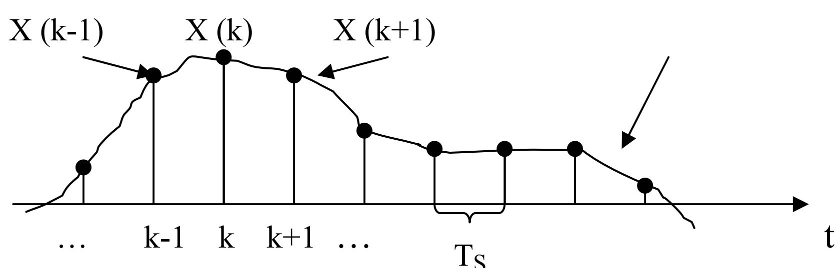


Figure 6.27 A continuous signal and its samples to explain the differential coding schemes.

In differential coding systems, the difference between the current sample $X(k)$ and its predicted estimate $\hat{X}(k)$ is encoded by PCM quantizers of the previous section, i.e., $d(k) = Y(k) = X(k) - \hat{X}(k)$.

The amplitude variations in this difference $Y(k)$ is expectedly significantly less that the original signal value and a lower mean-squared error. In turn, this results at a higher signal-to-quantizing noise ratio. Improvement can be either used for a higher quality transmission or the number of levels in the quantizer can be reduced to achieve a fixed SNR. Reduction in quantizer size corresponds to a reduction in the required system bandwidth. In the system block diagram of Figure 6.28, a linear minimum mean-square optimized predictor is used (the development of such predictor is outside the scope of this text.). In order this technique to work properly, the decoder has an identical copy of the prediction logic.

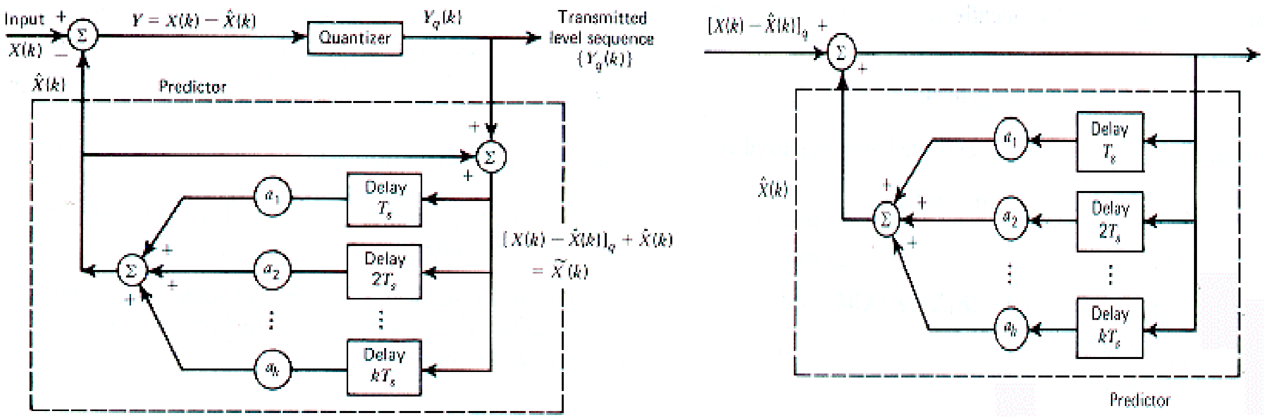


Figure 6.28 Differential Quantizing Techniques. Transmitter and receiver Circuits.

In the optimized differential system of Figure 6.28, the difference obtained as described above:

$$d(k) = Y(k) = X(k) - \hat{X}(k). \tag{6.30}$$

is quantized and transmitted. In order this technique to work properly, the decoder has an identical copy of the prediction logic.

$$\hat{X}(k) = a_1.\hat{X}(k-1) + a_2.\hat{X}(k-2) + \cdots + a_n.\hat{X}(k-n) \tag{6.31}$$

where n is the order of predictor, i.e., the number taps in the feedback loop. We can re-write it as:

$$\hat{X}(k) = \hat{X}(k) + [X(k) - \hat{X}(k)]_q \tag{6.32}$$

subscript q in the last expression represents the quantized version of the difference signal. The coefficients $\{a_1, a_2, \cdots, a_n\}$ are commonly known as the linear predictor coefficients (LPC) in the community and they are obtained from the signal using well-established fast algorithms, such as Levinson recursion method based on sample autocorrelations.

At the receiver the recovered or equalized signal is presented the synthesis loop of the predictor as shown in Figure 6.28 and the sum is integrated by a low-pass filter to perform a D/A conversion. The output of this low-pass filter/interpolator is a replica of the original signal plus the unavoidable quantizing noise, which is significantly smaller than its counterpart in PCM systems.

It has been shown in literature that the SDR performance of DPCM systems along the line of PCM can be formulated as:

$$SDR_{dB} = \left.\frac{S_0}{N_q}\right|_{db} = 6.02 * n + \alpha \qquad where \quad -3 < \alpha < 15 \quad for\,speech \tag{6.33}$$

and n is the number of quantizing bits. Similar number range for the parameter α exists for other signals, which is very much signal-dependent. For the same SNR, DPCM would require 2 or 3 fewer bits per sample than the companded PCM and this is the reason why telephone DPCM systems often operate at a bit rate of R=32 kbits/s or less instead of the standard 64 kbits/s for companded PCM. Thus, it is possible to have twice more phone channels in a single T-1 frame.

Yet, another performance measure is the SDR improvement due to prediction or commonly known as the prediction gain G_P :

$$G_P \; dB = 10\log_{10}(\frac{P_m}{P_d}) \tag{6.34}$$

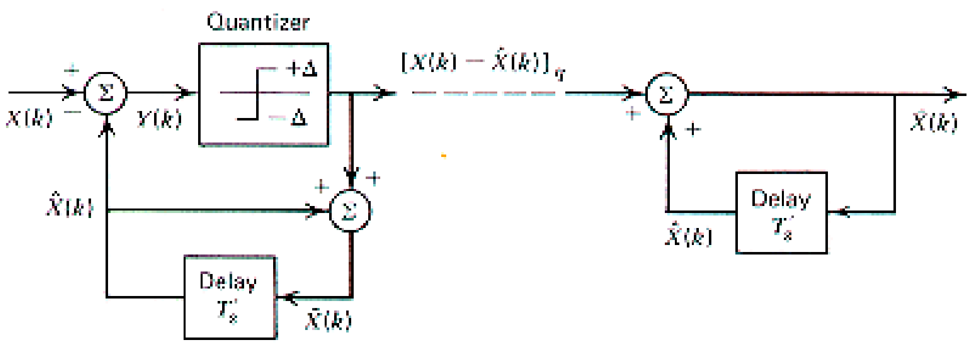
where P_m and P_d are the powers of $x(t)$ and $d(k) = Y(k)$, at time $t = kT_S$ respectively. Depending upon the order of terms used in prediction and the statistics of the input signal, i.e., speech, image, video, data, etc., the SDR improvement can be up to 25 dB over a PCM system.

6.7 Delta Modulation (DM) System

Delta Modulation is a one-bit oversampled A/D converter with a feedback loop. As in the conventional PCM, each code is a binary representation of both the sign and the magnitude of a particular sample. However, in DM there is only one bit to send for every sample, that is, the transmitted bit indicates whether that sample is larger or smaller than the previous one. From that perspective DM can be treated as a one-bit DPCM system. In this case, the fundamental question will be:

How one can achieve the goal of reliably representing the original analog signal?

The answer lies in the sampling rate. If we recall that PCM systems use a sampler operating at the Nyquist rate of the incoming signal. Here though, the sampling rate is many times the Nyquist rate, which is often 8 times or more. The functional block diagram of a DM system is shown in Figure 6.29.



(a) Transmitter (b) Receiver

Figure 6.29 Functional block diagram of a linear DM system.

At the transmitter the sample is compared with a predicted value and the difference is quantized into either Δ or $-\Delta$, for which only one bit is needed to represent. At the receiver, the decoded values of the difference signal is added to the immediately preceding output of the receiver. The governing equations for this simple setup are:

$$\hat{X}(k) = \tilde{X}(k-1) \tag{6.35}$$

where $\tilde{X}(k-1)$ is the receiver output at time: $t = (k-1)T_S$ and

$$\tilde{X}(k) = \hat{X}(k) + [X(k) - \hat{X}(k)]_q = \tilde{X}(k-1) \pm \Delta \tag{6.36}$$

Again the subscript q denotes the quantized version of the quantity. The delay element and the adder in Figure 6.29 can be replaced by an integrator whose input is an impulse sequence of period T_S and the amplitude or a single step size $\pm\Delta$. The DM with an integrator is known as the single-integrating DM system. In order to improve the tracking capability a double-integration is also used in practice. An implementation of an integrating DM is shown in Figure 6.30.

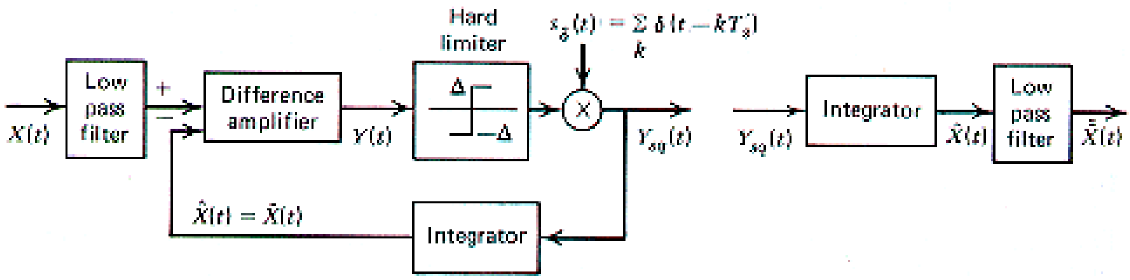


Figure 6.30 Implementation of an integrating DM system.

The stepwise operation of DM with increments, i.e., step-size $\pm\Delta$ at the transmitter and the receiver results in the waveforms shown in Figure 6.31. The process of going up on the staircase takes place as long as signal is above the predictor value, which is known as the start-up stage. It goes down when the reverse is true. If the staircase cannot quickly cross-over signal to track it closely we have a degradation called *slope overload noise* and this region is classified as the slope overload region. Easiest solution to this problem is to increase the sampling rate. In most practical case, this is not permitted due to the channel bandwidth restrictions. A third case is the oscillation between two directions when the two curves are very close to each other, which is known as the hunting region. Here the difference between the signal and its approximation is classified as the *granular noise*. If this quantity is large then it degrades the performance of the system.

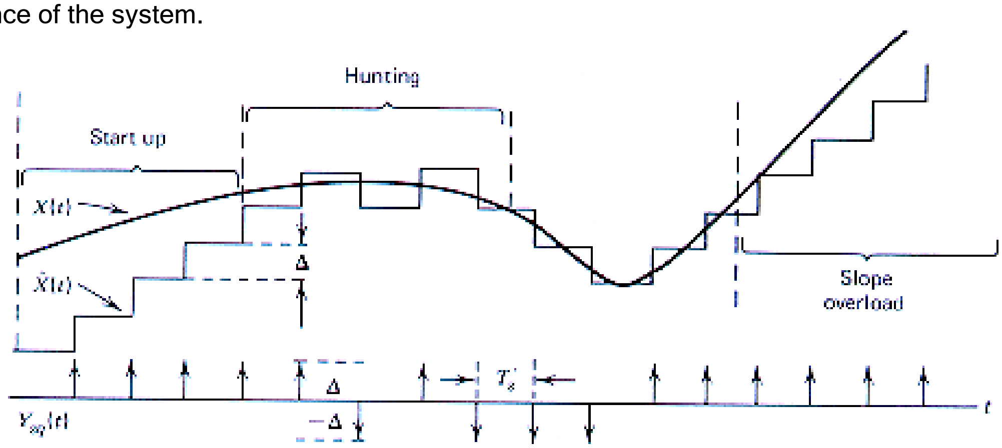


Figure 6.31 Delta Modulation signal and its staircase approximation.

The remedy to the second noise is to use a smaller step-size. But this will make it more difficult to combat the slope overload noise.

There are two routes people follow:

(1) to use the best step-size to avoid the slope overload condition and
(2) to include an adaptation scheme to change the step size up or down to minimize the error in quantization.

6.7.1 Slope Overload Condition: Large discrepancy occurs between the input signal and the DM output when the input signal varies more rapidly than the accumulator can closely track it. In other words, the step size Δ is too small for the system to follow the rapid changes in the input waveform. This pushes the system into the slope overload region with serious consequences. We now attempt to derive an expression for no overload. If $dx(t)/dt$ and f_S are the slope of the signal and the sampling rate, respectively, then there will be no overload if:

$$\left|\frac{d(x(t))}{dt}\right|_{\max} = 2\pi f_r A_{\max} < \Delta . f_S \qquad (6.37)$$

This relationship needs some clarification. If we had a single sinusoid as the input signal then $A_{\max}$ and f_r would be the peak magnitude of the signal and the frequency value at this peak, respectively.

However, in real-life applications we do not deal with single sinusoids, instead the input signal has much more complex structure, such as speech, image, radar, etc.

In practice, we use a representative or equivalent frequency number to have an idea where when the overload condition occurs rather than an exact number. For instance, in speech coding applications with a nominal bandwidth of 4,000 Hz, it is experimentally determined that the peak amplitude occurs in the neighborhood of $f_r = 800\ Hz$ and $w \approx 2\pi.800 = 1{,}600\pi.$. For this choice we compute the maximum speech input level to be:

$$A_{\max}^{voice} \approx \frac{\Delta.f_S}{2\pi.800} \tag{6.38}$$

6.7.2 Performance of DM Systems:

Suppose that the slope overload condition is avoided by selecting a step size Δ satisfying the above equations or by an increase in the sampling rate. Then the performance is affected by the granular noise. Let us assume that the granular noise is distributed uniformly in the range $\pm\Delta$ shown in the Figure 6.32.

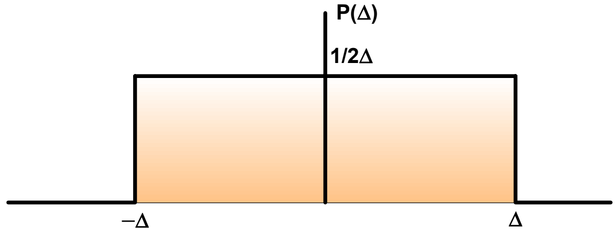


Figure 6.32 Uniform distribution of noise

The power content of the noise is given by:

$$\overline{\varepsilon}^2 = \frac{1}{2\Delta}\int_{-\Delta}^{\Delta}\varepsilon^2 d\varepsilon = \frac{\Delta^2}{3} \tag{6.39}$$

If we have a bandwidth B and the sampling rate is f_S then the noise power is equal to:

$$N_0 = \frac{\Delta^2}{3}.\frac{B}{f_S} = \frac{\Delta^2 B}{3 f_S} \tag{6.40}$$

On the other hand, if the signal power is $S_0 = E\{x^2(t)\}$ watts, where E stands for statistical average. The signal-to-distortion ratio for no slope overload condition is equal to:

$$\frac{S_0}{N_0} = \frac{3 f_s E\{x^2(t)\}}{\Delta^2 B} \tag{6.41}$$

As we did in the PCM case, let us assume that the signal peak is m_P occurring at a frequency f_r and the bandwidth is B Hz and substitute: $m_P = \Delta.f_S / 2\pi.f_r$ into the last equation to get:

$$\frac{S_0}{N_0} = \frac{3 f_S^3 E\{x^2(t)\}}{4\pi^2 B.m_P^2} \tag{6.42}$$

For instance, if the input is speech with a bandwidth 4,000 Hz and $f_r = 800\ Hz$ we get:

$$\left.\frac{S_0}{N_0}\right|_{dB} = 10\log_{10}\{\frac{150}{\pi^2}.\left(\frac{f_S/2}{B}\right)^3.\frac{E\{x^2(t)\}}{m_P^2}\} \tag{6.43}$$

Example 6.8: Design an audio DM system with at least 30 dB performance. Let us assume the following:

- Audio signal has 4.0 kHz bandwidth with an approximate representative frequency: $f_r = 800\ Hz$ and Peak amplitude of 10 Volts.
- Average to peak power ratio, which is defined by: $E\{x^2(t)\}/m_P^2$ is equal to 1/2.
- $f_r = 800\ Hz$

a. If we substitute given parameters into (6.43) and solve it for the required sampling frequency, we obtain: $f_S = 40.7\ kbits/s$. But for implementation considerations, we would be choosing $f_S = 48\ kbits/s$.

b. Let us find the minimum value of the 1-bit quantizer step size Δ to avoid the slope overload condition:

$$2\pi f_r A_{\max} < \Delta.f_S \quad \Rightarrow \quad \Delta \geq \frac{2\pi f_r A_{\max}}{f_S} = \frac{2\pi.800.10}{48{,}000} = \frac{\pi}{3} \approx 1.05\ Volts$$

c. On the other hand, the audio bandwidth of 4,000 Hz corresponds to a Nyquist frequency of 8,000 bits/second. Hence, the over-sampling factor should be at least 6 times the Nyquist rate in order to achieve the expected performance.

d. If we were to compare this with an equivalent PCM system, it would turn out that $R = (2B)n$ and $5 \leq n \leq 6$ bits. In this case, the average SDR for PCM system would be $30.1 \leq SNR_{PCM} \leq 36.1$ dB. Thus, under these conditions, the PCM system with a comparable bandwidth would perform almost same as this DM system with a 5, 6 times over sampling. It turns out that we have exchanged the sampling rate for the coding rate. If a cost-economical high-speed sampling device is available then a DM with simple low-rate coder can be the choice. However, if a high-rate sampling is not available then a PCM with a more complex coder is selected.

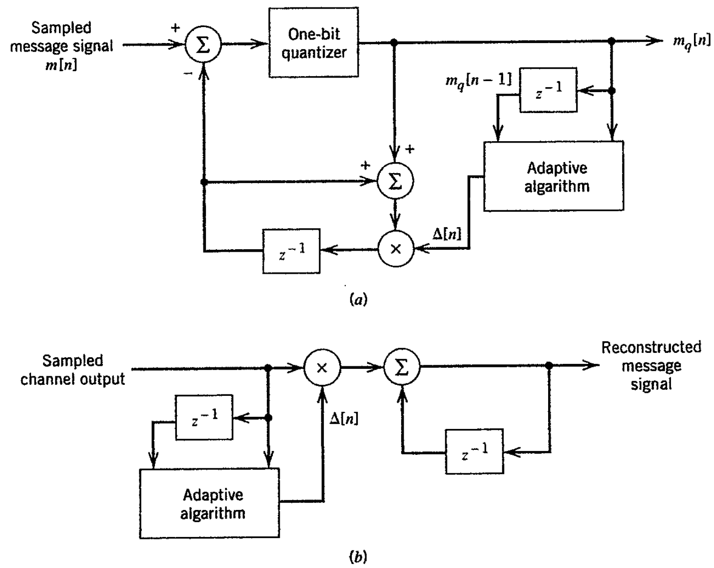
Example 6.9: Design an "Adaptive Delta Modulation" system, where step-sizes are adjusted according to a predetermined logic described by:

1. If successive difference signals are of opposite polarity, then DM is in "Granular Mode" and it is advantageous to reduce the step-size by a factor.
2. If successive difference signals are of same polarity, then DM is in "Slope-Overload Mode" and step size is to be increased by another factor.

The schematic for this type of ADM is shown in Figure 6.33. The following algorithm based on the principle of reducing/increasing step-size by 50% at each iteration of the adaptation process can be formulated by:

$$\Delta[n] = \begin{cases} \dfrac{\left|\Delta[n-1]\right|}{m_q[n]}(m_q[n] + 0.5m_q[n-1] & if\ \Delta[n-1] \geq \Delta_{\min} \\ \Delta_{\min} & if\ \Delta[n-1] < \Delta_{\min} \end{cases} \tag{6.44}$$

where $\Delta[n]$ is the step-size at iteration n and the quantizer output is $m_q[n] = \pm 1$.



Adaptive delta modulation system: (*a*) Transmitter. (*b*) Receiver.

Figure 6.33. Transmitter and receiver blocks for the adaptive DM system in Example 6.9.

In the Appendix D, we have chosen the following specific values for the Matlab implementation:

Signal: $m(t) = A.Sin(2\pi \ f_m t)$

Amplitude: $A = 1$

Sampling frequency: $f_S = 100 * f_m$

Adaptation threshold: $\Delta_{\min} = 1/8$

Following can be observed from the Matlab implementation:

- Because of the adaptivity of step-size, the ADM system tracks the changes in the input signal much better than Linear DM.
- Improved tracking performance of ADM results in an output signal with a much lower bit rate on the average.

6.8 Performance Comparison of Advanced Pulse Modulation Systems

Finally, we present in Figure 6.34 a comparative plot of the three systems used in speech waveforms and in pixel-domain image compression. Highest performance for comparable rate is obtained in the DPCM system with a feedback adaptation for its quantizer step-size. The next one is an Adaptive Delta modulation with a one-bit memory, and the final system is a logarithmically companded PCM. It is worth noting that the DM and LOG-PCM curves have a crossover point, which is around 48 kbits/s. This implies that a simple logarithmic PCM will do a better job than the equivalent DM and no need to worry about the feedback mechanism and the propagation of bit errors in the latter one.

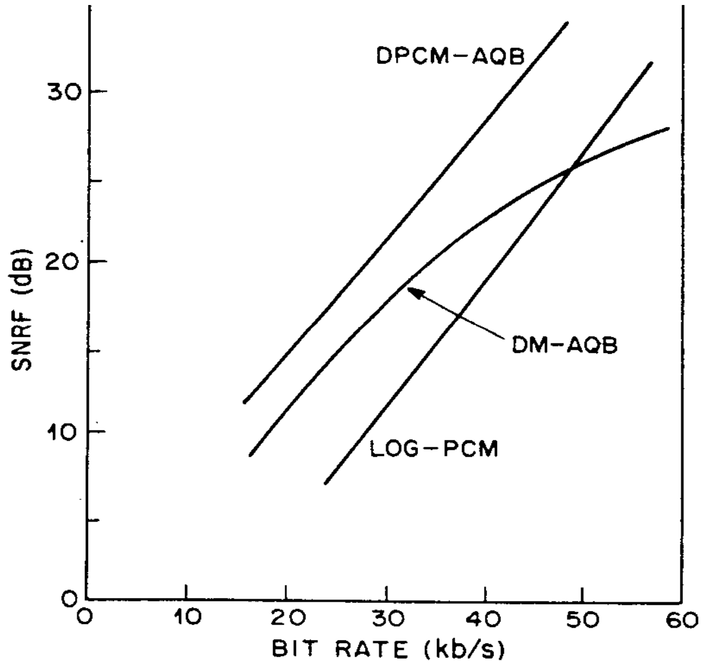


Figure 6.34 Comparative performance curves for PCM, DPCM and DM systems. (Reprint from Modern Analog and Digital Communication Systems, Third Edition, B. Lathi, courtesy of Oxford Press)