

Chapter 5. ANGLE MODULATION SYSTEMS

Information carrying message signal $x(t)$ is loaded into the carrier frequency or the phase of the transmitted signal rather than the amplitude in these systems. In these systems, the amplitude is constant at all times.

5.1 Instantaneous Phase and Frequency

Basic question is how do we make a sinusoid vary its frequency over time? To demonstrate that let us consider the following signal:

$$x_c(t) = A_c \cdot \text{Cos}(w_c t + \phi(t)) = \text{Re}\{A_c \cdot e^{j(w_c t + \phi(t))}\} = A_c \cdot \text{Re}\{e^{j\theta_c(t)}\} \quad (5.1)$$

where “Re” denotes the real part of a complex function and $\theta_c(t)$ is the total instantaneous phase angle of the modulated signal $x_c(t)$ is defined as

$$\theta_c(t) = w_c t + \phi(t) \quad (5.2)$$

Similarly, the instantaneous frequency of $x_c(t)$ is the derivative:

$$w(t) \equiv \frac{d\theta_c}{dt} = w_c + \frac{d\phi}{dt} \quad (5.3a)$$

$$f(t) \equiv \frac{1}{2\pi} \frac{d\theta_c}{dt} = f_c + \frac{1}{2\pi} \frac{d\phi}{dt} \quad (5.3b)$$

The functions $\phi(t)$ and $\frac{1}{2\pi} \frac{d\phi}{dt}$ are known as the “Phase Deviation” and the “Frequency Deviation,” respectively. Depending on the nature of the relationship between $\phi(t)$ and the signal $x(t)$, two types of angle modulation are known in the literature:

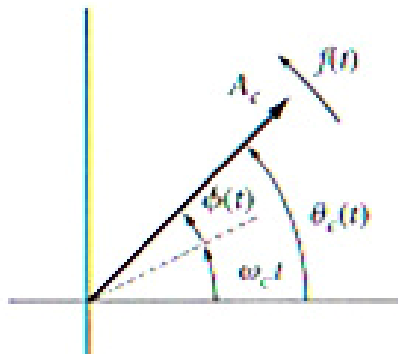


Figure 5.1 Rotating phasor representation of FM/PM modulation (Carlson 5.1-1)

- Phase Modulation (PM), where the phase deviation of the carrier is proportional to the modulating information signal:

$$\phi(t) = \phi_{\Delta} \cdot x(t) = k_p \cdot x(t) \quad (5.4a)$$

and

$$x_c(t) = A_c \cdot \text{Cos}(w_c t + \phi_{\Delta} \cdot x(t)) \quad (5.4b)$$

where $k_p = \phi_\Delta$ is the “**phase deviation constant**” or “**PM modulation index**” in radians/unit of $x(t)$.

- Frequency Modulation (FM) implies that the frequency deviation of the carrier is proportional to the modulating signal:

$$\frac{d}{dt} \phi(t) = 2\pi \cdot f_\Delta \cdot x(t) = k_F \cdot x(t) \quad \text{resulting in} \quad f(t) \equiv f_c + f_\Delta \cdot x(t) = f_c + k_f \cdot x(t) \quad (5.5a)$$

and

$$x_c(t) = A_c \cdot \text{Cos}[w_c t + 2\pi \cdot f_\Delta \int_t x(\lambda) \cdot d\lambda] \quad (5.5b)$$

where $k_f = 2\pi \cdot f_\Delta$ is the **FM modulation index**.

The phase deviation of a frequency-modulated carrier at a given time is given by the following integral:

$$\frac{d}{dt} \phi(t) = k_f \cdot \int_{t_0}^t x(\alpha) d\alpha + \phi_0 \quad (5.6)$$

in which ϕ_0 is the phase deviation at $t = t_0$. It follows from (5.6) that k_F is measured in radians per second per unit of $x(t)$. Since, it is often more convenient to measure frequency deviation in hertz (Hz), we express it as follows:

$$k_F = 2\pi f_d \quad (5.7)$$

where f_d is known as the “**frequency-deviation constant**” of the modulator in hertz per unit of $x(t)$.

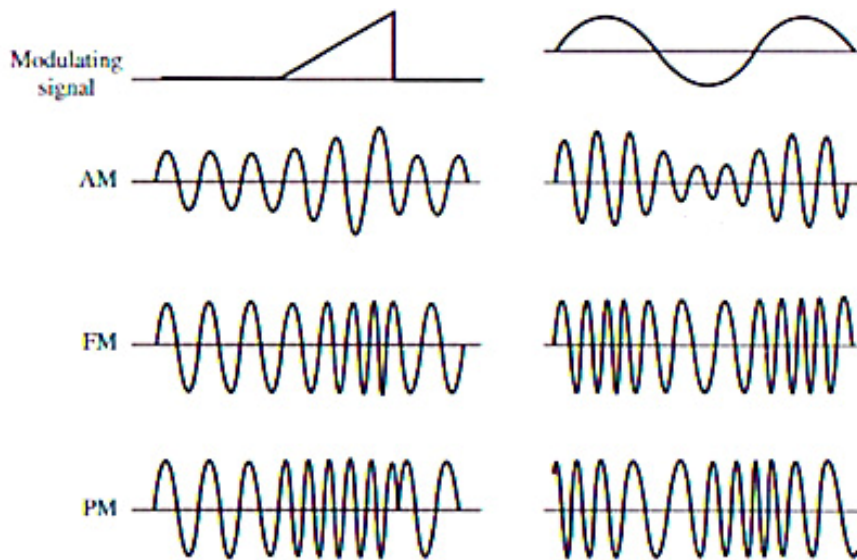
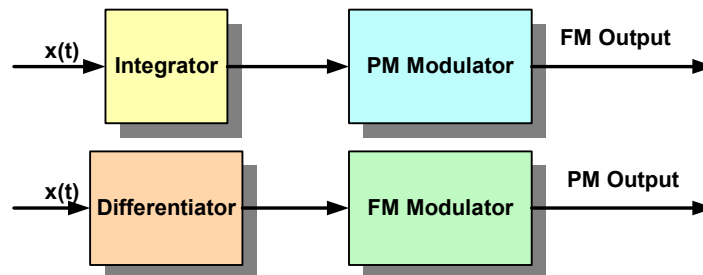


Figure 5.2 Time-domain output signals from AM, FM, and PM modulators for a triangular pulse and a sinusoidal pulse. In the case of AM the signal shape is explicit as an envelope. But the signal is buried in the frequency or phase content of waveforms in FM and PM modulators. In FM, the frequency of the output varies with the signal, whereas, there are 180 degree phase-shifts in PM whenever signal changes its sign.

With these definitions, the modulator output for these two cases can be written in a compact form:

$$x_c(t) = \begin{cases} A_c \cdot \text{Cos}[w_c t + k_p x(t)] & \text{for PM} \\ A_c \cdot \text{Cos}[w_c t + 2\pi \cdot f_d \int_t x(\alpha) d\alpha] & \text{for FM} \end{cases} \quad (5.8)$$

where the actual limits of integral is not specified. From (5.8), we see that the instantaneous frequency of the PM modulator output is f_c for both $t < t_0$ and $t > t_0$. On the other hand, phase is advanced by $k_p = \pi/2$ radians for $t > t_0$ resulting in a discontinuous signal at crossover point. The phase of the FM modulator output is continuous but the frequency is advanced to $f_c + f_d$ for $t > t_0$. Careful observation of (5.8) indicates that PM and FM are closely related. An FM signal can be generated by first integrating $x(t)$ and then using the result as an input to the PM modulator. Conversely, a PM signal can be obtained from an FM waveform via differentiation. Hence, these two modulation schemes can be easily interchanged as shown in below.



Both FM and PM Modulators can be configured to generate the other modulation output using an integrator or a differentiator stage as pre-processor.

Example 5.1¹: Generate and sketch FM and PM waveforms for a triangular signal $m(t)$ as shown on the left with $k_f = 2\pi \cdot 10^5$ and $k_p = 10\pi$ where $f_c = 100$ MHz. (Note that the derivative of a triangular pulse train is a rectangular pulse train as shown on the right.)

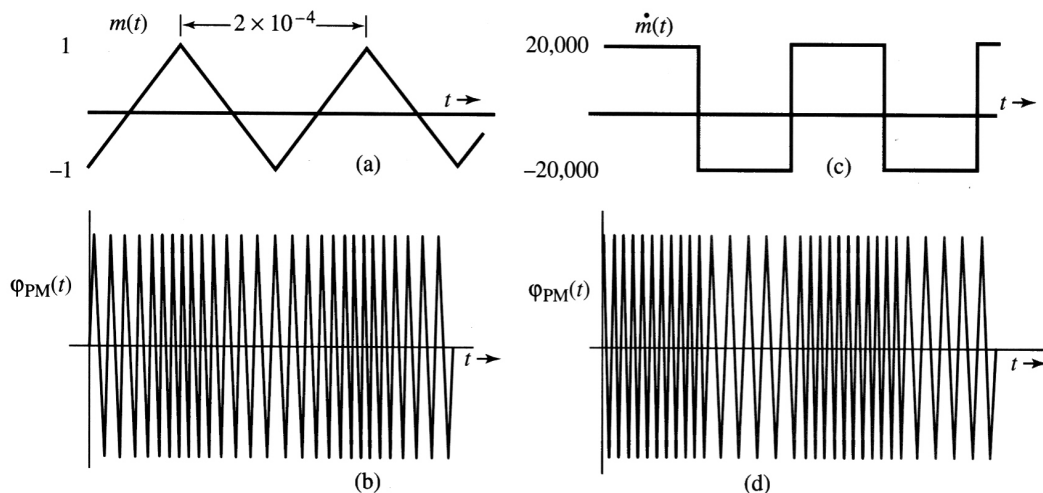


Figure 5.3 FM and PM Modulators for a triangular signal (Courtesy of B. Lathi, Oxford Press)

¹ This problem is Courtesy of B. Lathi from his text, Oxford Press. It is worth noting that Lathi is using $m(t)$ for signal not $x(t)$.

Analysis of FM:

$$\frac{\omega_i}{2\pi} = \frac{\omega_c}{2\pi} + \frac{k_f m(t)}{2\pi} \quad \text{which yields} \quad f_i = f_c + \frac{k_f}{2\pi} m(t) = 10^8 + 10^5 m(t) \quad (5.9)$$

$$(f_i)_{\min} = 10^8 + 10^5 \cdot [m(t)]_{\min} = 99.9 \text{ MHz} \quad \text{and} \quad (f_i)_{\max} = 10^8 + 10^5 \cdot [m(t)]_{\max} = 100.1 \text{ MHz}$$

It is easy to observe that as $m(t)$ goes up or down linearly with time, so does w_i for FM.

Analysis of PM:

$$f_i = f_c + \frac{k_p}{2\pi} \dot{m}(t) = 10^8 + 5\dot{m}(t) \quad (5.10)$$

$$(f_i)_{\min} = 10^8 + 5 \cdot [\dot{m}(t)]_{\min} = 10^8 - 10^5 = 99.9 \text{ MHz}$$

$$(f_i)_{\max} = 10^8 + 5 \cdot [\dot{m}(t)]_{\max} = 10^8 + 10^5 = 100.1 \text{ MHz}$$

Here, as $\dot{m}(t)$ switches between $\{\pm 1\}$, so does ω_i change between its minimum and maximum values.

5.2 Power Content and Bandwidth of Angle Modulation Systems

As in the case of various amplitude modulation (AM) schemes, the system performance is measured primarily in terms of the power dissipated and the bandwidth needed to transmit the message without any distortion, if possible. First, we will write the power equation and then attempt to present the bandwidth concept after classifying the angle modulation schemes according to their bandwidth needs.

5.2.1 Power: Power is equally critical in angle modulation schemes as it was the case for amplitude modulation schemes of the previous chapter. Observation of (5.8) implies that the amplitude of FM and PM waveforms is bound by $|x(t)| \leq A_c$, hence, the total power is equal to:

$$P = (1/2) \cdot A_c^2 \quad \text{Watts.} \quad (5.11)$$

5.2.2 Bandwidth: Bandwidth is a fairly complicated concept for angle modulation systems due to the non-linear equations involved. Almost all the time, it is computed after the system under consideration is classified as a narrow-band or a wide-band system. With this difficulty in mind, let us express the angle modulated signal (generic for both PM and FM) in an exponential form:

$$x_c(t) = A_c e^{j[\omega_c t + \phi(t)]} = A_c \cdot e^{j\omega_c t} \cdot e^{j\phi(t)} \quad (5.12)$$

This is called the analytic signal representation:

$$x_c(t) = \text{Re}\{x_c(t)\} = \text{Re}\{A_c \cdot e^{j\omega_c t} \cdot e^{j\phi(t)}\} \quad (5.13)$$

where $\text{Re}(\cdot)$ implies the real part of the argument. Next, let us expand the second exponential term $e^{j\phi(t)}$ in (5.16) in a power series to have:

$$x_c(t) = \text{Re}\{A_c \cdot e^{j\omega_c t} \cdot [1 + j\phi(t) - \frac{1}{2!}\phi^2(t) + \dots + j^n \frac{1}{n!}\phi^n(t) + \dots]\} \quad (5.14)$$

$$x_c(t) = A_c \cdot \underbrace{\cos\omega_c t}_{\text{carrier}} - A_c \cdot \underbrace{\phi(t) \cdot \sin\omega_c t}_{AM1} - \frac{1}{2!} A_c \cdot \underbrace{\phi^2(t) \cdot \sin\omega_c t}_{AM2} + \frac{1}{3!} A_c \cdot \underbrace{\phi^3(t) \cdot \sin\omega_c t}_{AM3} + \dots$$

Observation Notes:

1. The generalized angle modulation (PM and FM) has a carrier signal plus infinitely many AM modulated terms. Therefore, if the original input signal $x(t)$ has a bandwidth $W \text{ Hz}$ or $2\pi W \text{ radians/s}$, then, $\phi(t)$ will have a bandwidth: $W \text{ Hz}$.

2. Bandwidth of the second-order term, i.e., that of $\phi^2(t)$ will be $2W$ Hz. If we extend this treatment to n^{th} order term, then the spectrum of $\phi^n(t)$ will be nW Hz.
3. Due to the infinitely many terms in (5.14), it is easy to see that FM and PM systems have infinite bandwidth requirement. Luckily, most power resides in a few terms.

5.3 Narrow-Band FM and PM

If the modulation index k_f is small, such that $|k_f a(t)| \ll 1$, or equivalently, $|\phi(t)|_{\max} \ll 1$ then the higher-order terms in (5.17) are negligible and the FM waveform can be approximated by:

$$\begin{aligned} x_{FM}(t) &\cong A_c \cdot \cos \omega_c t - A_c \phi(t) \sin \omega_c t \\ &= A_c \cdot \cos \omega_c t - A_c \cdot k_f a(t) \sin \omega_c t \end{aligned} \quad (5.15)$$

This is reminiscent of the AM systems of Chapter 4, where the output contains an explicit carrier component and a term in which is proportional to $x(t)$. But the modulation term has a 90° phase-shift due to the sine term. This multiplication generates a pair of sidebands around carrier frequency $\pm \omega_c$ as shown in Figure 5.4. Since $x(t)$ is band limited to B Hz, $\phi(t)$ is also band limited to $2\pi W$ and the bandwidth of the output signal $x_{FM}(t)$ will be $4\pi W$ radians/s. This form of FM is called the Narrow Band FM (NBFM). It is easy to see that a narrow band PM signal is given by:

$$x_{PM}(t) \cong A[\cos \omega_c t - k_P x(t) \sin \omega_c t] \quad (5.16)$$

Equations (5.15) and (5.16) illustrate that FM and PM signals can be generated using DSB-SC modulators.

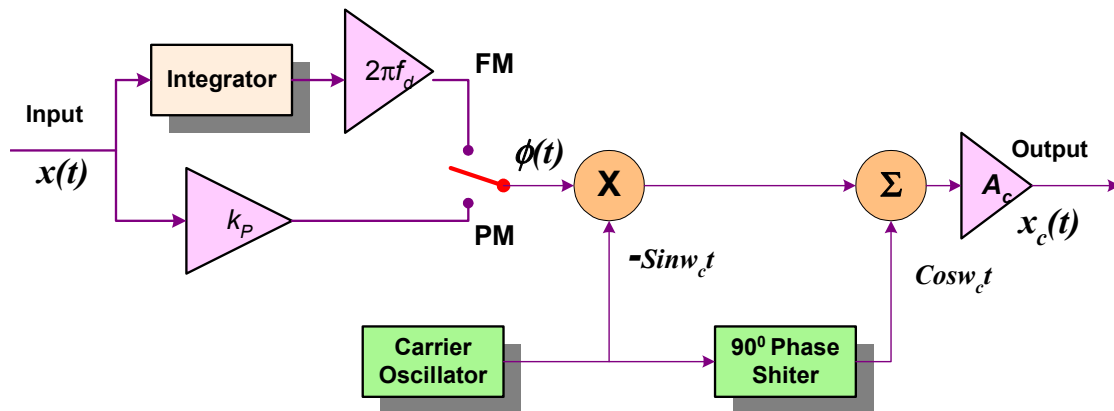


Figure 5.4 Generation of Narrow Band FM. Switch allows for generation of either NBFM or NBPM.

Example 5.2: Tone Modulation: Consider an FM system operating with a single tone sinusoid: Recalling from equation (5.6) with $t_0 = 0$

$$\phi(t) = k_f \cdot A \cdot \int_0^t \cos(\omega_m \alpha) d\alpha = (k_f \cdot A / \omega_m) \cdot \sin(\omega_m t) = (A f_d / f_m) \cdot \sin(\omega_m t) \quad (5.17)$$

Substituting this into the FM equation we obtain:

$$x_{FM}(t) = A_c \cdot \cos[\omega_c t + (A f_d / f_m) \cdot \sin \omega_m t] \quad (5.18)$$

If $f_d / f_m \ll 1$, the modulator output is narrow band and can be approximated by:

$$\begin{aligned}
 x_{FM}(t) &= A_c \cdot \text{Cos} w_c t - A_c \cdot (A f_d / f_m) \cdot \text{Sin} w_c t \cdot \text{Sin} w_m t \\
 &= A_c \cdot \text{Cos} w_c t + A_c / 2 \cdot (A f_d / f_m) \cdot [\text{Cos}(w_c + w_m)t - \text{Cos}(w_c - w_m)t]
 \end{aligned}
 \tag{5.19}$$

or in exponential form:

$$x_{FM}(t) = A_c \cdot \text{Re} \{ e^{j w_c t} [1 + (2 A f_d / f_m) \cdot (e^{j w_m t} - e^{-j w_m t})] \} \tag{5.20}$$

The spectrum has carrier plus two side bands around it as clearly marked in Figure 5.5.

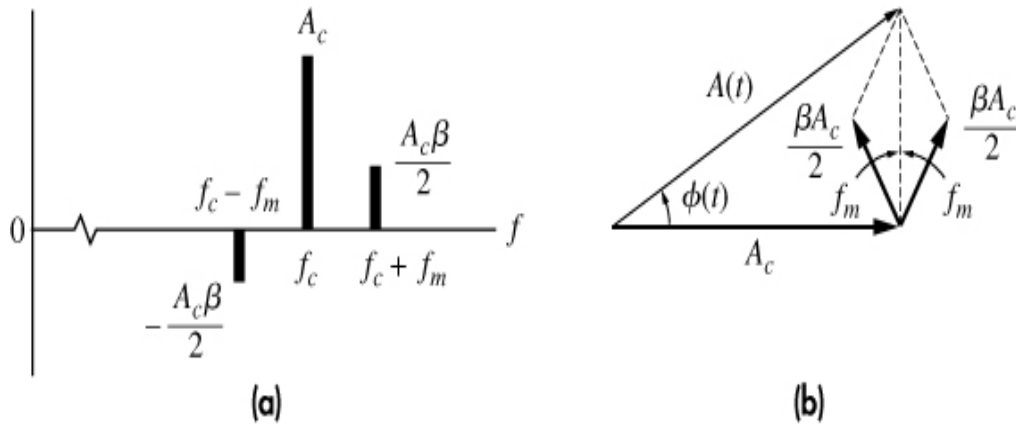


Figure 5.5 Narrow Band FM Tone Modulation. (a) Line spectrum and (b) phasor diagram.

5.4 Wide-Band FM/PM

If the modulation index k_f is not small enough to satisfy the constraint $|k_f a(t)| \ll 1$ or equivalently, $|\phi(t)|_{\max} \approx 1$ then we cannot ignore the higher-order terms in (5.14). In this case, to study a WBFM (WBPM) system or commercial FM (PM), we need to resort to some practical approximation techniques. This can be demonstrated by a tone modulation of a single sinusoid with frequency f_m ; i.e., $\text{Cos}(w_m t)$.

$$x_{FM}(t) = A_c \cos \left[\omega_c t + k_f \int_{-\infty}^t x(\alpha) d\alpha \right] \tag{5.21}$$

when the sinusoid is used as the input it becomes:

$$x_{FM}(t) = A_c \text{Cos} [w_c t + k_f \int_{-\infty}^t \text{Cos}(w_m \alpha) d\alpha] = A_c \text{Cos} [w_c t + \beta \cdot \text{Sin}(w_m t)] \tag{5.22}$$

Here, β is the modulation index and represents the maximum value of phase deviation for FM and PM.

$$\beta = k_f \cdot A_m / w_m \tag{5.23}$$

The output signal can be expressed in terms of exponential terms as before:

$$\begin{aligned}
 x_{FM}(t) &= A_c \cdot \text{Re} \{ e^{j 2\pi f_c t} \cdot e^{j \beta \cdot \text{Sin}(2\pi f_m t)} \} \\
 &= A_c \cdot \text{Re} \{ e^{j w_c t} \cdot e^{j \beta \cdot \text{Sin} w_m t} \}
 \end{aligned}
 \tag{5.24}$$

The function $e^{j\beta \cdot \text{Sin}w_m t}$ is periodic with a period w_m . This form does not resemble any of the functions we have seen earlier. As we have done for other periodic functions, we can expand that in a Fourier series:

$$x_{FM}(t) = A_c \cdot J_0(\beta) \cdot \text{Cos}w_c t + A_c \sum_{n=\text{odd}}^{\infty} J_n(\beta) [\text{Cos}(w_c + nw_m)t - \text{Cos}(w_c - nw_m)t] \\ + A_c \sum_{n=\text{even}}^{\infty} J_n(\beta) [\text{Cos}(w_c + nw_m)t + \text{Cos}(w_c - nw_m)t] \quad (5.25)$$

where the coefficients are Bessel functions of the first kind, order 0, argument β .

5.4.1 Bessel Functions: Bessel functions cannot be evaluated in a closed form. On the other, it has been well studied in applied mathematics. The integral is a function of n and β . It is known as the Bessel Function of the first kind of order n and argument β and denoted by $J_n(\beta)$ as tabulated in a table and plotted in Figure 5.6 together with the first six Bessel terms.

$n \backslash \beta$	0.2	0.5	1.0	2.0	5.0	8.0
0	.990	.938	.765	.224	-.178	.172
1	.100	.242	.440	.577	-.328	.235
2	.005	.031	.115	.353	.047	-.113
3			.020	.129	.365	-.291
4			.002	.034	.391	-.105
5				.007	.261	.186
6					.131	.338
7					.053	.321
8					.018	.223

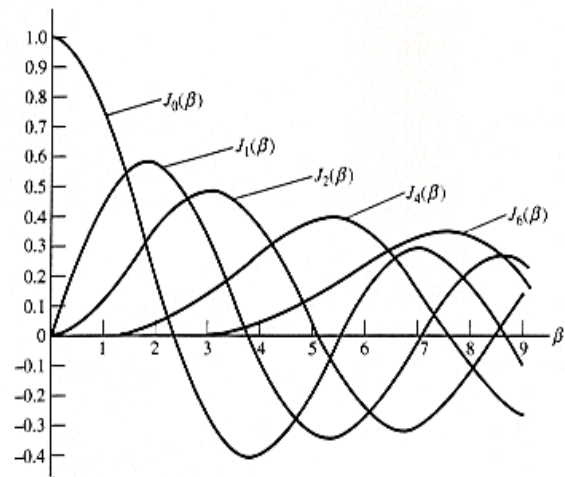
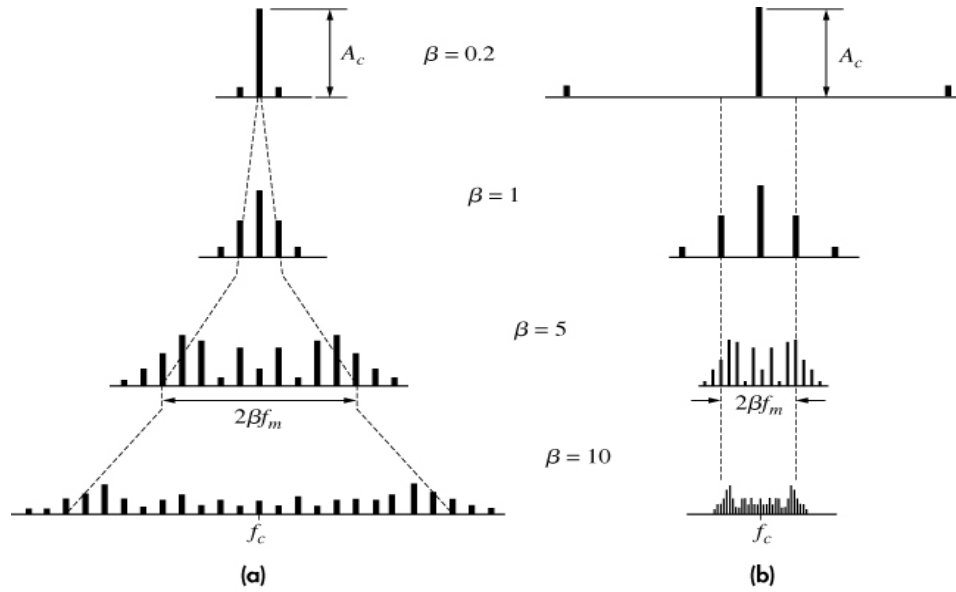


Figure 5.6 Bessel Function values and actual curves as described by (5.28a-5.28f) and below the tone modulation for (a) FM and (b) PM with f_m fixed. (Carlson 5.1-7)



With the aid of Bessel functions, the Fourier series expansion can be written as:

$$e^{j\beta \cdot \sin w_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot e^{jn w_m t} \quad (5.26)$$

to permit the modulator output to be:

$$x_c(t) = A_c \cdot \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot \cos(w_c + n w_m)t \quad (5.27)$$

The spectrum has components at the carrier frequency and has an infinite number of sidebands at integer multiples of the modulating signal frequency w_m . The amplitude of each spectral sideband can be determined either from the table above or the plots. A few properties of Bessel functions will be appropriate at this point.

5.4.2 Properties Bessel Functions:

1. For n even, we have $J_n(\beta) = J_{-n}(\beta)$. (5.28a)

2. For n odd, we have $J_n(\beta) = (-1) J_{-n}(\beta)$. (5.28b)

3. For small values of β , we can use the following approximations:

$$J_0(\beta) \cong 1 \quad (5.28c)$$

$$J_1(\beta) \cong \beta/2 \quad (5.28d)$$

$$J_n(\beta) \cong 0 \quad n > 2 \quad (5.28e)$$

$$\sum_{n=-\infty}^{n=+\infty} J_n^2(\beta) = 1 \quad (5.28f)$$

4. The above analysis is valid for PM as well and we need to write the modulation index in terms of PM notation:

$$\beta = k_p A_m \quad (5.29)$$

5.4.3 Power Content and SNR for Wideband FM Modulation: When the signal is modulated to generate an FM signal, the power in the side frequencies may appear only at the expense of the power originally in the carrier, thereby making the amplitude of the carrier dependent on β . Note that the average power of an FM signal is given by:

$$P_c = \frac{A_c^2}{2} \sum_{n=-\infty}^{n=+\infty} J_n^2(\beta) = \frac{A_c^2}{2} \quad (5.30)$$

This is due to the fourth property of the Bessel function presented above. This can be interpreted as the average power across 1-Ohm resistor is constant as it should be.

The SNR for modulated signals, when there is no pre-emphasis and de-emphasis filtering to improve the performance of commercial FM systems is given by:

$$\left(\frac{S}{N}\right)_o = \begin{cases} \frac{P_M \beta_P^2}{[\text{Max}|x(t)|]^2} \cdot \frac{P_R}{N_0 W} & \text{for PM} \\ 3 \frac{P_M \beta_F^2}{[\text{Max}|x(t)|]^2} \cdot \frac{P_R}{N_0 W} & \text{for FM} \end{cases} \quad (5.31)$$

where $\text{Max}|x(t)|$ denotes the maximum magnitude of the message signal, P_M is the power budget of the message signal, P_R stands for the power in the received signal, W is the bandwidth of the message signal and N_0 is the noise power density level.

When there are pre-emphasis and de-emphasis filters with a 3-dB cutoff frequency f_0 , the improved signal-to-noise ratio (SNR) is given by:

$$\left(\frac{S}{N}\right)_{oPD} = \frac{(W/f_0)_3}{3[B/f_0 - \arctan(W/f_0)]} \cdot \left(\frac{S}{N}\right)_o \quad (5.32)$$

Let us recall that the SNR values are normally measured in terms of decibells and the values obtained in (5.31) and (5.32) need to be converted to dB units.

Example 5.3: (Carlson Example 5.1-2) Consider a NBFM signal:

$$x_c(t) = 100 \cdot \text{Cos}(2\pi \cdot 5000t + 0.05 \cdot \text{Sin}2\pi \cdot 200t)$$

Instantaneous frequency:

$$f(t) = \frac{1}{2\pi} \frac{d\theta}{dt} = \frac{1}{2\pi} \cdot [2\pi \cdot 5000t + 0.05 \cdot (400\pi) \cdot \text{Cos}2\pi \cdot 200t] = 5000 + 10 \cdot \text{Cos}2\pi \cdot 200t$$

which results in $f_c = 5000 \text{ Hz}$, $f_\Delta = 10$, and the information signal modulated: $x(t) = \text{Cos}400\pi t$.

For NBFM tone modulation, the form for the phase term is:

$$\phi(t) = \beta \cdot \text{Sin}w_m t \text{ from above we see that } \beta = 0.05$$

The line-spectrum is exactly as shown in Figure 5.4 (Carlson 5.1-4) with:

$$A_c = 100; \text{sidelobes: } A_c \beta / 2 = 2.5$$

The transmitted power becomes:

$$S_T = 1/2 \cdot [(-2.5)^2 + 100^2 + (2.5)^2] = 5006.25$$

$$S_T^P = 1/2 \cdot (100)^2 = 5000$$

Therefore, only 6.25 watts of amplitude distortion is observed due to the sidelobes.

5.4.4 Bandwidth of Wideband FM Modulation and Carson's Rule: Let us now compute the bandwidth for WBFM. To compute bandwidth, we need to determine how many sideband lines are to be included. All sidebands with relative amplitude satisfying:

$$|J_n(\beta)| > \epsilon \quad (5.33a)$$

are called significant sidebands. Here $0.01 \leq \epsilon \leq 0.1$. It is easy to see from the properties of Bessel functions:

$$\text{If } |J_M(\beta)| > \epsilon \text{ then } |J_{M+1}(\beta)| < \epsilon \quad (5.33b)$$

then the FM bandwidth can be written as a function of significant terms $M(\beta)$

$$B_{FM} = 2.M(\beta).f_m = 2M(D).W$$

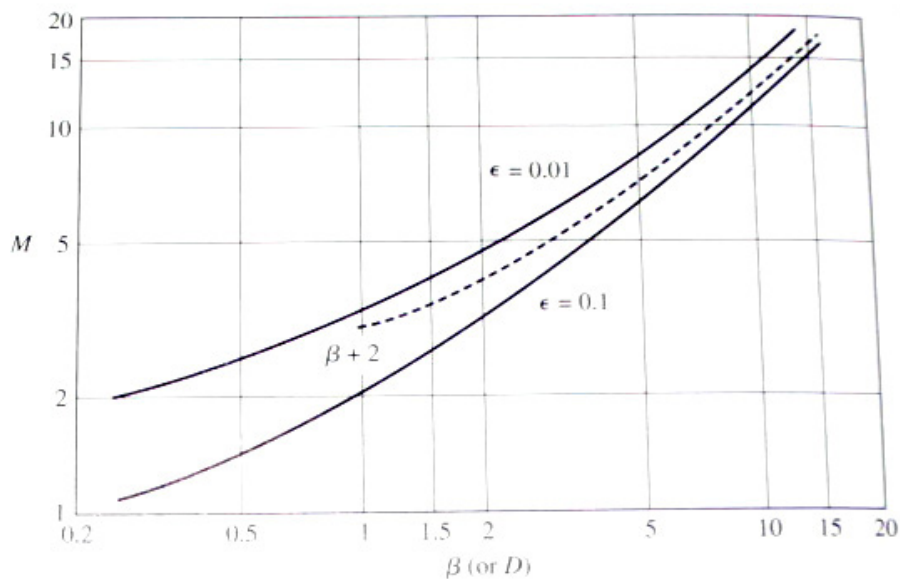


Figure 5.7 Number of significant sideband pairs as a function of β or D .

As shown in Figure 5.7 (Carlson 5.2-1), upper and lower limit curves for ϵ are plotted. From these figures for a given modulation index β or equivalently deviation ratio defined as $D = f_\Delta / W$. For many practical cases and approximation of

$$M(\beta) \approx \beta + 2 \quad (5.33c)$$

is used, which is also shown the figure.

Using these two equations we get the FM bandwidth:

$$B_{FM} \approx 2.(\beta + 2).f_m = 2.\left(\frac{A_m f_\Delta}{f_m} + 2\right).f_m = 2.(A_m f_\Delta + 2f_m) \quad (5.34a)$$

which is known as the Carson's Rule in the literature. For a generic signal with bandwidth W , we need to reformulate the above expression as:

$$B_{FM} \approx 2.(f_\Delta + W) = 2(D + 1).W \quad \text{for } D \gg 1 \text{ or } D \ll 1 \quad (5.34b)$$

However, significant majority of FM systems have $2 < D < 10$, which require a modification to the Carson's Rule:

$$B_{FM} \approx 2.(f_{\Delta} + 2.W) = 2(D + 2).W \quad \text{for } D > 2 \quad (5.35)$$

Example 5.4: In North America, the maximum value of frequency deviation standard is $\Delta f = 75 \text{ kHz}$, and the adjacent FM stations must be separated by a minimum of 200 kHz. Let us use the baseband audio signal range as $W = 30 \text{ Hz} - 15 \text{ kHz}$, typical value used for radio broadcast, the corresponding value of the deviation ratio is:

$$D = \frac{75 \text{ kHz}}{15 \text{ kHz}} = 5$$

Equation (5.35) yields $B_{FM} = 2(5 + 2).15,000 = 210 \text{ kHz}$, which has an 10 kHz over estimate, whereas Carson's Rule of (5.34b) results in $B_{FM} = 180 \text{ kHz}$, which is an under estimate by 20 kHz.

Example 5.5: Consider the system with an FM modulator with output: $x_{FM}(t) = 100.Cos(2\pi 1000t + \phi(t))$ where the signal is: $x(t) = 5.Cos(2\pi.8t)$

In other words, $f_m = 8$. The modulator is followed by a band-pass filter centered at 1000 Hz with a bandwidth of 56.0 Hz. From Bessel function calculations various terms are listed below.

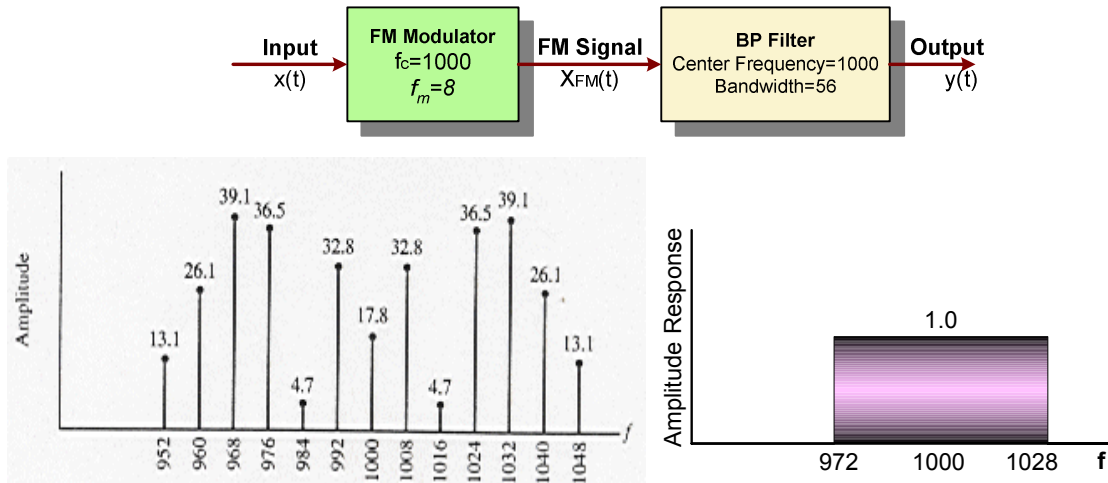


Figure 5.8 Block diagram of the FM system in Example 5.4, the significant components and the spectrum.

Now let us find the power of the output signal. Peak deviation: $5.f_m = 40 \text{ Hz}$, which results in $\beta = 40/8 = 5$. For this set of parameters we need on the order of 16 sideband terms (both sides combined) from Figure 5.7. In Figure 5.8, we have displayed the amplitudes of significant terms as well as the filter characteristics. From the frequency readings we conclude that the BP filter permits the component at the carrier frequency and three significant terms on each side. The normalized power for these seven terms will be:

$$P_7 = J_0^2(5) + 2.[J_1^2(5) + J_2^2(5) + J_3^2(5)] = 0.178^2 + 2.[0.328^2 + 0.047^2 + 0.365^2] = 0.518$$

The power before the BP filter at the output of the modulator stage is:

$$P_T = \frac{1}{2} A_c^2 = \frac{1}{2} 100^2 = 5000 \text{ Watts}$$

and the power at the output of the filter is simply:

$$P_o = 5000 * 0.518 = 2589 \text{ Watts}.$$

5.4.5 Bandwidth of Wideband PM Modulation and Carson's Rule: If we repeat the previous analysis for the WBPM case, we get:

$$B_{PM} = 2.M(\phi_{\Delta}).W \approx 2.(\phi_{\Delta} + 1).W \quad (5.36a)$$

In terms of phase modulation index, Carson's Rule for the bandwidth of WBPM can be re-written as:

$$B_{PM} = 2(\Delta f + W) = 2\left(\frac{k_P.m'_P}{2\pi} + W\right) \quad \text{where } m'_P = |\dot{x}(t)_{\min}| = |\dot{x}(t)_{\max}| \quad (5.36b)$$

It is important to observe that B_{PM} depends directly on the maximum/minimum value of the derivative of the message signal $x(t)$.

Summary of Features of Angle Modulated Systems:

1. One can trade power for BW.
2. It is possible to trade SNR for BW (Deep space uses large BW \Rightarrow low power)
3. They have better immunity to non-linearities.

(Skip the section on Linear Distortion, a simplified version will be presented in the next section for non-linear distortion.)

5.5 FM Wave Generation

There are two basic methods for generating FM waveforms: The indirect method, frequently known as the Armstrong technique and the direct method, which uses either the PLL and VCO setup as in the SSB generation or the Hartley oscillator.

5.5.1 Armstrong Indirect Method: It is a combination of a NBFM modulator followed by a set of frequency multipliers. For instance, if it is required to achieve a 12-fold increase in the frequency deviation, we can either use an 12th-order nonlinear device or two 2nd-order and one 3rd-order devices in tandem. From the simplicity perspective, the latter will be unquestionably the designers' choice. It is worth noting that the output needs to be band-limited with a bandpass filter centered at $12\omega_c$. Here, the carrier frequency as well as the frequency deviation f_{Δ} are 12 times the original values.

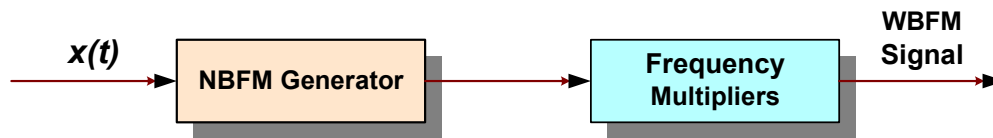


Figure 5.9 Block diagram of an FM generation using Armstrong Method.

The FM generated in this fashion as depicted in Figure 5.9 has both amplitude and the phase distortion. The amplitude distortion is attributed to exclusion of the non-linear AM terms in:

$$X_{c_{FM}}(t) = A \left[\underbrace{\cos \omega_c t - k_f a(t) \sin \omega_c t}_{\text{Terms useful for FM}} - \underbrace{\frac{k_f^2}{2!} a^2(t) \cos \omega_c t + \dots}_{\text{Unwanted nonlinear AM Terms}} \right] \quad (5.37)$$

The amplitude distortion can be eliminated by a hard-limiter, but the phase distortion, as we will consider below, is very critical in all practical applications and it has to be properly handled.

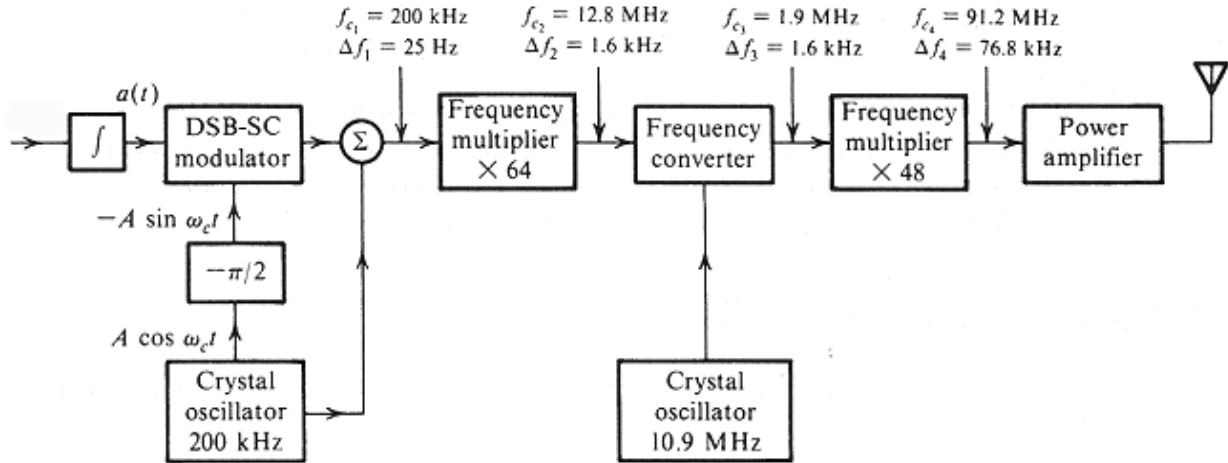


Figure 5.10 Functional diagram of the FM generator in Example 5.6 (Courtesy of Lathi, Oxford Press)

Example 5.6: Let us demonstrate the overall FM waveform generation for a commercial radio station operating at $f_c = 91.2$ MHz with the industry standard $\Delta f = 75$ kHz. As it is clear from (5.37), the unwanted nonlinear AM signal needs to be cancelled, which is normally done by a hard-limiting device.

Design steps:

1. As shown in Figure 5.10, for NBFM, let us choose $f_{c_1} = 200$ kHz and $\Delta f_1 = 25$ Hz, which results in a small modulation index, $\beta \ll 1$.
2. Let us assume that the audio signal has a frequency range in the interval: 50 Hz – 15 kHz (hi-fi), then β ranges in the interval: $\Delta f / f_m = 25 / 50 = 0.5$ and $25 / 15,000 = 0.00167 \ll 1$ as desired in a NBFM case.
3. To achieve $\Delta f = 75,000$ Hz, we need a multiplication factor of $75,000 / 25 = 3,000$. This multiplication factor is very difficult to achieve with ordinary amplifiers. In practice, we design these systems with a set of cascades of 2nd order and 3rd order non-linearities (e.g., diodes and RLC components) then we use multipliers of 64 and 48 resulting in:

$$\Delta f_4 = 64 * 48 * \Delta f_1 = 3,072 * 25 = 76,800 \text{ Hz.}$$

5.5.2 Phase Distortion in FM Generators Based on Armstrong Indirect Method: Let us approximate the NBFM wave of (5.41) after neglecting the nonlinear terms:

$$x_{FM}(t) = A[\cos \omega_c t - k_f a(t) \sin \omega_c t] = AE(t) \cos[\omega_c t + \theta(t)] \quad (5.38)$$

If we use the maximum values of 1.0 for both the cosine and sine terms, the amplitude and phase components will be given by:

$$E(t) = \sqrt{1 + k_f^2 a^2(t)} \quad (5.39a)$$



$$\theta(t) = \tan^{-1}[k_f a(t)] \quad (5.39b)$$

Assuming the amplitude distortion in the envelope term $E(t)$ is either negligibly small or eliminated by a hard-limiter, the distortion in phase, or equivalently, distortion in frequency is found by concentrating on (5.39b) alone. In this case, let us now find the instantaneous frequency by differentiating (5.39b):

$$w_i(t) = \dot{\theta}(t) = \frac{k_f \dot{a}(t)}{1 + k_f^2 a^2(t)} = \frac{k_f x(t)}{1 + k_f^2 a^2(t)} \quad (5.40)$$

This equation is not very useful in this form. As we have done in many similar situations let us write the power series expansion and analyze each of the resulting series:

$$\begin{aligned}
 w_i(t) &= k_f \cdot x(t) \cdot [1 - k_f^2 \cdot a^2(t) + k_f^4 \cdot a^4(t) - \dots] \\
 &= k_f \cdot x(t) - k_f \cdot x(t) [k_f^2 \cdot a^2(t) - k_f^4 \cdot a^4(t) + \dots]
 \end{aligned} \tag{5.41}$$

 **Desired term**
 **Nonlinear Frequency / Phase Distortions**

Nonlinear terms (2^{nd} , 3^{rd} , ...) in (5.41) are integral part of the instantaneous frequency equation and they cannot be removed.

To study their significance of let us use the single-tone modulation case. Assume a message signal oscillating with the frequency: $f_m \ll f_c$ and $m(t) = 2 \cdot \text{Cos} \omega_m t$. Here the term going into the FM generation equation, the modulation index, and the instantaneous frequency will be:

$$\begin{aligned}
 a(t) &= \frac{2 \cdot \text{Sin} \omega_m t}{\omega_m} \\
 \omega_i(t) &= \beta \omega_m \cdot (1 - \beta^2 \sin^2 \omega_m t + \beta^4 \sin^4 \omega_m t - \dots) \cdot \text{Cos} \omega_m t
 \end{aligned}$$

When compared to the first two terms, the term $\beta^4 \sin^4 \omega_m t$ and subsequent ones are very small and they can be neglected. The resulting instantaneous frequency is now approximated as:

$$\omega_i(t) \cong \beta \omega_m \cdot (1 - \beta^2 \text{Sin}^2 \omega_m t) \text{Cos} \omega_m t.$$

Using half-angle formula from trigonometry:

$$\beta^2 \text{Sin}^2 \omega_m t = (\beta^2 / 2) \cdot (1 - \text{Cos} 2\omega_m t)$$

Let us use this in the last expression:

$$\begin{aligned}
 \omega_i(t) &\cong \beta \omega_m \cdot [\text{Cos} \omega_m t - \frac{\beta^2}{2} \cdot \text{Cos} \omega_m t + \frac{\beta^2}{2} \cdot \text{Cos} \omega_m t \cdot \text{Cos} 2\omega_m t] \\
 &\qquad\qquad\qquad \underbrace{\hspace{10em}} \\
 &\qquad\qquad\qquad \frac{1}{2} \cdot \frac{\beta^2}{2} \cdot (\text{Cos} \omega_m t + \text{Cos} 3\omega_m t)
 \end{aligned}$$

$$\begin{aligned}
 w_i(t) &= \beta \omega_m \cdot [(1 - \frac{\beta^2}{2} + \frac{\beta^2}{4}) \cdot \text{Cos} \omega_m t + \frac{\beta^2}{4} \text{Cos}(3\omega_m t)] \\
 &= \beta \omega_m \cdot [(1 - \frac{\beta^2}{2}) \cdot \text{Cos} \omega_m t + \frac{\beta^2}{4} \cdot \text{Cos}(3\omega_m t)]
 \end{aligned}$$

For small values of modulation index, i.e., $\beta \ll 1$, the first term is insignificant and can be neglected. The resulting instantaneous frequency is now given by:

$$\begin{aligned}
 w_i(t) &\cong \beta \omega_m \cdot \text{Cos} \omega_m t + \frac{\beta^3 \omega_m}{4} \cdot \text{Cos} 3\omega_m t
 \end{aligned} \tag{5.42}$$

 **Desired Frequency**
 **3rd-Harmonic Distortion**

In other words, we have to be concerned with the third harmonic distortion. In practice, this distortion is normally expressed as the percentage of the 3rd-Harmonic with respect to the desired fundamental oscillation frequency:

$$\% 3^{\text{rd}} \text{ Harmonic Distortion} = \frac{\beta^3 w_m / 4}{\beta w_m} = \frac{\beta^2}{4} \quad (5.43)$$

As a specific numerical example let us compute the 3rd-harmonic distortion for the Armstrong modulator of Example 5.6. Recall that the modulation index was changing in the range:

$$0.00167 < \beta < 0.5.$$

Using the largest value of 0.5 we get a 3rd-Harmonic Distortion level of :

$$\%3^{\text{rd}} - \text{HarmonicDistortion} = 1/16 = 0.0625 = 6.25\%.$$

5.5.3 WBFM Generation Using Direct Method: In this class, the WBFM waveform is generated by a Voltage Controlled Oscillator (VCO) whose controlling voltage comes from an external source and the oscillator is usually a Colpitts or a Hartley oscillator.

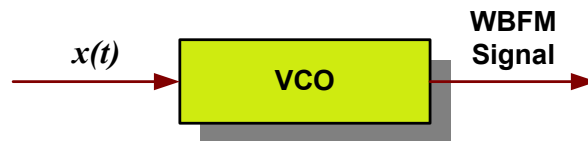


Figure 5.11 Block diagram wideband FM generation using VCO.

As an example to FM wave generation via direct method, let us study the following Colpitts Oscillator tuned circuit:

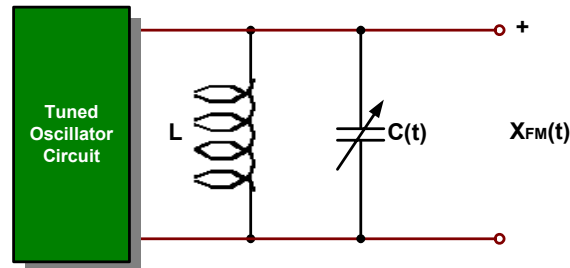


Figure 5.12 Colpitts Oscillator for FM generation by direct method.

The natural resonance frequency of this circuit is $\omega_0 = 1/\sqrt{LC}$. Let us vary the capacitor as a function of the modulating signal, $x(t)$, i.e., $C = C_0 - k.x(t)$ and substitute it into the natural frequency expression:

$$\omega_0 = \frac{1}{\sqrt{LC_0[1 - k.x(t)/C_0]}} = \frac{1}{\sqrt{LC_0} \cdot [1 - k.x(t)/C_0]^{1/2}} \quad (5.44)$$

The square root operation in the denominator can be eliminated by utilizing the binomial approximation:

$$(1 + x)^n \cong 1 + nx \quad \text{if } |x| \ll 1$$

$$\omega_0 \approx \frac{1}{\sqrt{LC_0}} \cdot \left[1 + \frac{k.x(t)}{2C_0}\right] \quad \text{for } \frac{k.x(t)}{2C_0} \ll 1 \quad (5.45)$$

Natural Resonance Frequency: The last result can be written as a generic oscillation equation and recognize terms on both sides of the equality:

$$\omega_0 = \omega_c + k_f .x(t) = \omega_c \cdot \left[1 + \frac{k.x(t)}{2C_0}\right] \quad (5.46)$$

In other words:

$$\omega_c = 1/\sqrt{LC_0} \quad \text{and} \quad k_f = k.\omega_c / 2.C_0 \quad (5.47)$$

The maximum Capacitance Deviation in the tuned circuit is given by:

$$\Delta C = km_p = \frac{2k_f C_0 m_p}{\omega_c} \quad (5.48)$$

where m_p is the peak value of the modulating signal. The normalized capacitance deviation is a more commonly known term and it is defined as:

$$\frac{\Delta C}{C_0} = \frac{2k_f m_p}{\omega_c} = \frac{2\Delta f}{f_c} \quad \text{if } \frac{\Delta f}{f_c} \ll 1 \quad (5.49)$$

In all practical applications Δf is significantly smaller than the carrier frequency f_c , then the normalized capacitance deviation $\Delta C / C_0$ is a very small quantity.

Observations:

1. Direct FM produces sufficient frequency deviation and requires little frequency multiplication as the work horse for budget FM radio transmission.
2. It has poor frequency stability without the use of feedback stabilization. In practice, some form of feedback mechanism is used to increase the stability.

5.6 Demodulation of Angle-Modulated Signals

5.6.1 FM Demodulation with Frequency Discriminator: An FM demodulator needs to produce an output voltage that is linearly varying with the frequency of the signal coming from the channel as shown in Figure 5.13. The systems that produce such response are called “*Frequency Discriminators*”.

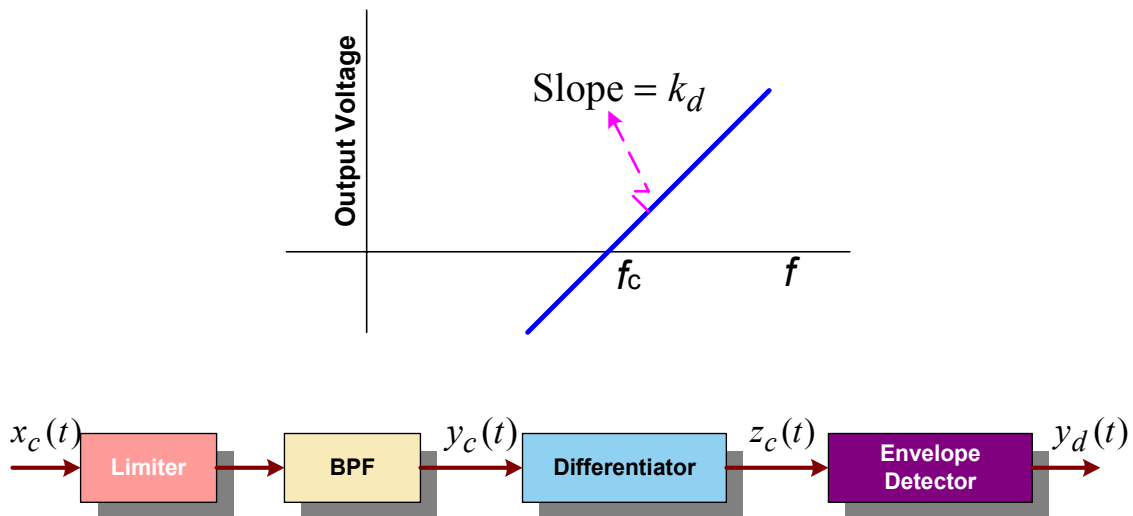


Figure 5.13 Frequency Discriminator and an FM Demodulation with Limiter.

If the input to the discriminator is an angle modulated signal:

$$x_c(t) = A_c \cdot \text{Cos}[w_c t + 2\pi f_d \cdot \int_t x(u) du] = A_c \cdot \text{Cos}[w_c t + k_d \cdot \int_t x(u) du] \quad (5.50)$$

where the discriminator output will be simply:

$$y_d(t) = k_d \cdot k_f \cdot x(t) \quad (5.51)$$

where k_d is the discriminator sensitivity with linear characteristics as shown. A fairly good approximation to the ideal linear characteristics can be obtained by the use of a differentiator followed by an envelope detector. The differentiator output is simply the derivative:

$$\begin{aligned}
z_c(t) &= \frac{dy_c(t)}{dt} \approx \frac{dx_c(t)}{dt} = \frac{d}{dt} \{A_c \cdot \text{Cos}[\omega_c t + k_f a(t)]\} \\
&= A_c \cdot [\omega_c + k_f \frac{d}{dt} a(t)] \cdot \text{Sin}[\omega_c t + k_f a(t)] \\
&= A_c \cdot [\omega_c + k_f x(t)] \cdot \text{Sin}[\omega_c t + k_f a(t)]
\end{aligned} \tag{5.52}$$

Since both the envelope term: $A_c[\omega_c + k_f x(t)]$ and the oscillatory term: $\text{sin}[\omega_c t + k_f a(t)]$ in the derivative signal of (5.52) are explicitly time-dependent, the result is a combination of amplitude and frequency modulation operations.

The envelope of term can be demodulated by means of an envelope detector introduced in the previous chapter. It is easy to see that $\Delta\omega = k_f m_p < \omega_c$, which results in $\omega_c + k_f x(t) > 0$ for all t .

In this analysis, the amplitude of the incoming FM wave is assumed to be constant at A_c . If it were a function of time, then there would be an additional term containing dA_c/dt , which would cause the final output to be proportional to: $A_c(t) \cdot x(t)$, then the recovery of the original information signal becomes impossible. Therefore, it is essential to maintain A_c as constant using a band-pass limiter circuit, which has been included in the system block diagram of Figure 5.14.

Example 5.7: Balanced Frequency Discriminator: In this case, we use a feedback system based on a pair of slope circuits with their transfer functions related by: $H(f_1) = H(-f_2)$ in the neighborhood of the carrier frequency f_c as plotted in Figure 5.14.

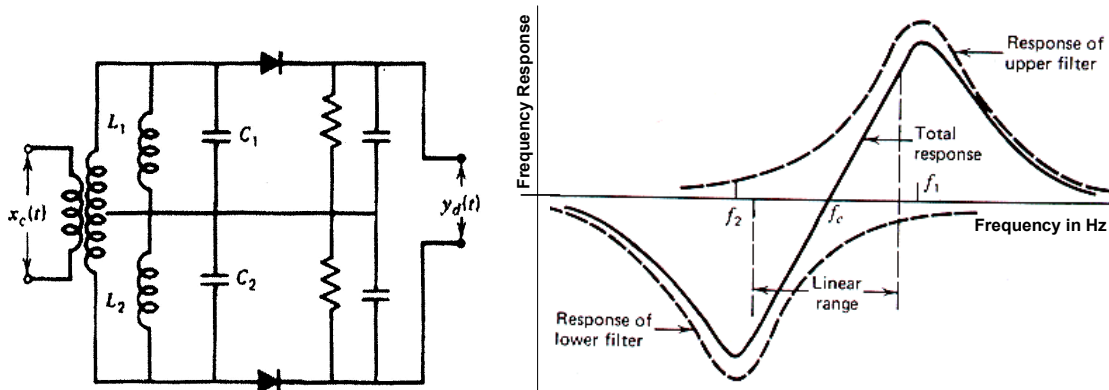


Figure 5.14 FM Demodulation by balanced frequency discriminator circuit and the frequency response.

The performance of this setup is better than discriminators in the presence of noise and hence they are the choice of designers. The center frequencies of the two band-pass filters are given by:

$$f_i = 1/2\pi\sqrt{L_i C_i} \tag{5.53}$$

The output of the upper envelope detector is usually approximated into the form:

$$|v_1(t)| \approx \pi B_{FM} a A_c \cdot [1 + \frac{2k_f}{B_{FM}} x(t)] \tag{5.54}$$

where the bias term $\pi B_{FM} a A_c$ is proportional to the slope a of the transfer function of the upper slope circuit. Note that B_{FM} is the FM transmission bandwidth of the FM system. Similarly, the output from the lower branch is approximated by:

$$|v_2(t)| \approx \pi \cdot B_{FM} \cdot a \cdot A_c \cdot \left[1 - \frac{2k_f}{B_{FM}} \cdot x(t)\right] \quad (5.55)$$

The overall output will be:

$$v_o(t) = |v_1(t)| - |v_2(t)| = \pi \cdot B_{FM} \cdot a \cdot A_c \cdot x(t) \quad (5.56)$$

It is clearly a scaled version of the information signal in the absence of channel degradation.

5.6.2 Demodulation of Angle-Modulated Signals with PLL: Phase Lock Loop (PLL) are widely used in present communication systems to track the phase and frequency of the carrier of a received signal because of their superior performance, ease of tuning and ease of implementation with inexpensive IC devices. Therefore, it is a very powerful device for demodulation of signals with suppressed carrier or with a low-power pilot tone.

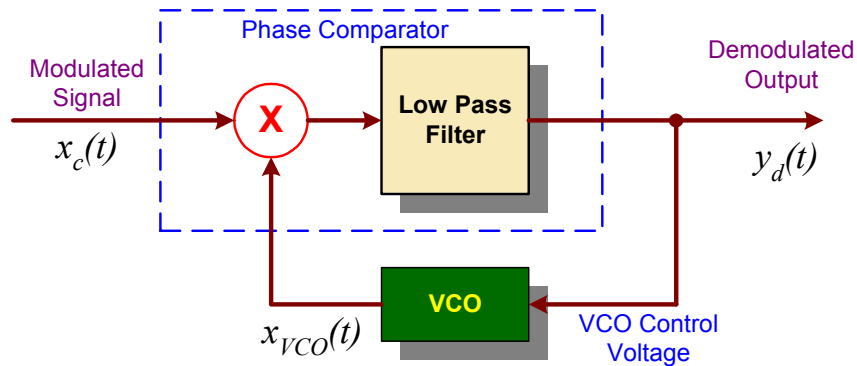


Figure 5.15 Demodulation of angle modulated signals by PLL.

PLL has three basic components:

- Voltage Controlled Oscillator (VCO),
- Multiplier or a phase comparator, and
- Loop Filter $H(w)$.

It is worth noting that the phase comparison is mathematically equivalent to the multiplication of two sinusoid voltages. The frequency of the voltage controlled oscillator (VCO) output varies linearly with its input voltage value and it is tuned to operate at a radian frequency w_c in the absence of any control voltage. When a control voltage of $y_d(t)$ is present, then the instantaneous frequency of the VCO is proportional to the control voltage. In other words,

$$w_{VCO} = w_c + g_{VCO} \cdot y_d(t) \quad (5.57)$$

where g_{VCO} is the sensitivity of the VCO and it is measured in terms of *radians/second/volt*. Using this frequency in the VCO output equation, we obtain:

$$\begin{aligned} w_{VCO} &= A_{VCO} \cdot \text{Cos}\left[w_c t + g_{VCO} \cdot \int_{-\infty}^t y_d(u) du\right] \\ &= A_{VCO} \cdot \text{Cos}\left[w_c t + g_{VCO} \cdot \theta_{VCO}(t)\right] \end{aligned} \quad (5.58)$$

where A_{VCO} is the VCO gain and u is a dummy integration variable. From Figure 5.15 loop, we can write the expression for the output of the phase comparator in the form:

$$y_d(t) = \frac{1}{2} \cdot A_c \cdot A_{VCO} \cdot \text{Sin}[\theta_i(t) - \theta_{VCO}(t)] \quad (5.59)$$

Under steady-state conditions, both the input and output sinusoids have the same frequency and the PLL is locked. In this case, the phase error $\theta_i(t) - \theta_{vco}(t)$ will be small to permit the small angle approximation to a sine function:

$$\text{Sin}[\theta_i(t) - \theta_{vco}(t)] \approx \theta_i(t) - \theta_{vco}(t) \quad (5.60)$$

This approximation allows the sinusoidal non-linearity to be neglected and the PLL becomes a linear feedback control system as shown in Figure 5.16.

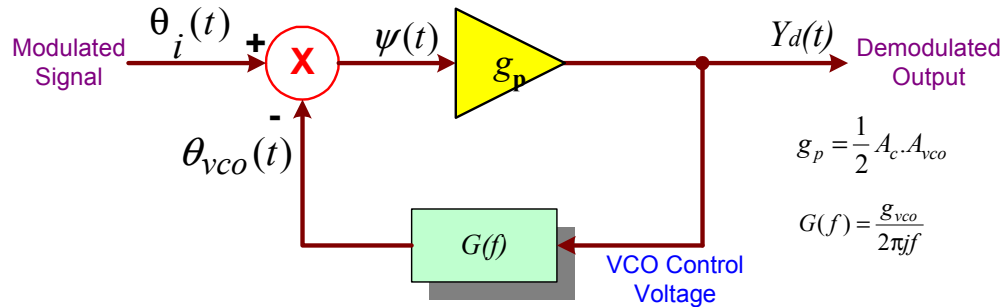


Figure 5.16 Linear PLL model for an FM demodulator.

5.6.3 Operation of a linear PLL as an FM Demodulator: Assume that the loop is in lock condition at $t = 0$ and a little later at $t = 0^+$ the frequency of the incoming FM signal changes by w , due to a change in the incoming signal (modulated signal in Figure 5.16). Then, at time $t = 0^+$, we have a phase difference of:

$$\psi(t) = \theta_i(t) - \theta_{vco}(t) \quad (5.61)$$

and its derivative will yield a frequency difference between the input frequency and the VCO frequency:

$$\frac{d}{dt} \psi(t) = \frac{d}{dt} [\theta_i(t) - \theta_{vco}(t)] \quad (5.62)$$

As $\psi(t)$ begins to increase, the phase comparator output will be positive to result of an increase in the frequency of the VCO. A new equilibrium point will be reached when the VCO frequency becomes equal to that of the input signal. Since the VCO frequency is proportional to $y_d(t)$, which, in turn, is the demodulated output. When the equilibrium is established, the frequency of the input signal and the VCO frequency will be identical; but, the phase difference will be no longer 0° . In order the VCO to operate at a frequency away from w_c , there must be a control voltage $y_d(t)$ that is generated by a phase difference other than 0° . It is not difficult to verify that, at equilibrium after a phase (frequency) change:

$$\theta_i(t) = wt; \quad \theta_{vco}(t) = wt - \frac{w}{g_p g_{vco}}; \quad \frac{d}{dt} \theta_i(t) = \frac{d}{dt} \theta_{vco}(t) \quad (5.63)$$

resulting in an output signal:

$$y_d(t) = w / g_{vco} \quad (5.64)$$

$$\psi(t) = \psi_e(t) = w / g_p \cdot g_{vco} \quad (5.65)$$

where $\psi_e(t)$ is the new equilibrium. When the frequency of the incoming FM signal changes continuously at a rate slower than the time required for the PLL to establish a new equilibrium, the PLL output will be continuously proportional the to the frequency deviation of the carrier, thus the system is tracking the signal very closely, i.e., we are working with a locked PLL. There are more precise models for PLL and its implementation as a FM modulator, which are beyond the scope of our subject in this text.

5.7 Interference in Angle-Modulation Systems

5.7.1 Interference: Let us consider an FM signal $A_c \cdot \text{Cos}(w_c t)$ is corrupted by interference I in the same frequency range as the carrier frequency: $I \cdot \text{Cos}[(w_c + w)t]$. Then received signal will be the sum of the transmitted signal plus this interfering signal:

$$\begin{aligned} x_r(t) &= A_c \cdot \text{Cos}(w_c t) + I \cdot \text{Cos}[(w_c + w)t] \\ &= (A_c + I \cdot \text{Cos}wt) \cdot \text{Cos}w_c t - I \cdot \text{Sin}wt \cdot \text{Sin}w_c t \\ &= E_r(t) \cdot \text{Cos}[w_c t + \psi_d(t)] \end{aligned} \quad (5.66)$$

where $E_r(t)$ is the envelope:

$$E_r(t) = \sqrt{(A_c + I \cdot \text{Cos}(wt))^2 + (I \cdot \text{Sin}wt)^2} \quad (5.67)$$

and the phase is given by:

$$\psi_d(t) = \tan^{-1} \left(\frac{I \cdot \text{Sin}wt}{A_c + I \cdot \text{Cos}wt} \right) \quad (5.68)$$

Suppose that the following approximations are valid:

$$I \ll A_c \quad \text{and} \quad \psi_d(t) \approx A_c + I \cdot \text{Sin}wt$$

Then the instantaneous frequency is given by:

$$w_i = w_c + \frac{d}{dt} \psi_d(t) = w_c + \dot{\psi}_d(t) \quad (5.69)$$

which yields a demodulator output for the FM case:

$$y_d(t) = \frac{I}{A_c} \text{Sin}wt \quad (5.70)$$

and similarly, an output for the PM case:

$$y_d(t) = \frac{I}{A_c} \text{Cos}wt \quad (5.71)$$

Both of these results are oscillating sinusoids. Hence, the recovery of the original signal $x(t)$ is not possible. If the interference is small: $I \ll A_c$, then the magnitude of the sinusoid becomes small and the output is dominated by the transmitted signal. In other words, FM demodulator suppresses weak interference completely.

As we have discussed in Chapters 1 and 2, all communication signals are corrupted by the channel noise as the signal travels from towards the intended receiver. In the case of FM and PM systems, the channel noise is, more appropriately assumed to be an additive white Gaussian noise with a flat spectrum. In practice, this is not the case. The effects of noise and its interpretation as interference in FM and PM systems are illustrated in Figure 5.17, which clearly demonstrates that the noise power in the receiver output is concentrated at higher frequencies. Audio and other information-bearing signals which has their dominant power spectral components in lower frequencies, this poses a difficult challenge. To combat that a pair of filters known as "Pre-emphasis and De-emphasis Filters" are often used.

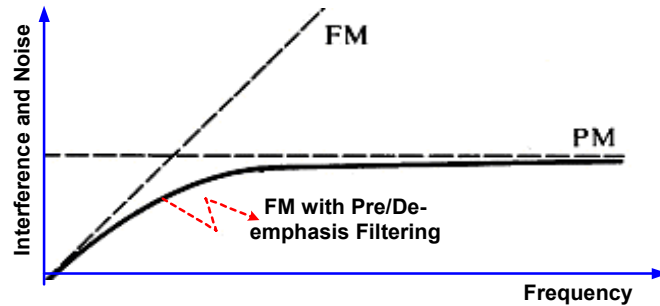


Figure 5.17 Effects of Noise on angle modulated systems and the FM with Pre/De-emphasis filtering.

5.7.2 Pre-emphasis and De-emphasis in FM for Spectral Shaping: Audio frequency spectrum used in FM radio transmission does not exhibit uniform levels across the commercial frequency range of 12.0 kHz. This is clearly observable in Figure 5.18, where a female speaker utters the word “information.” The power in the low-frequency components is dominant as seen from the spectrum. This is common since the power in human voice is concentrated between 400-2200 Hz and there is very little energy beyond 3.2 kHz. Can we throw away everything beyond that? The answer would be no except the ordinary voice communication over phone. This is due to the fact that the context, the prosody, the tonality, the speaker characteristics, language, etc, are spread all over the spectrum.

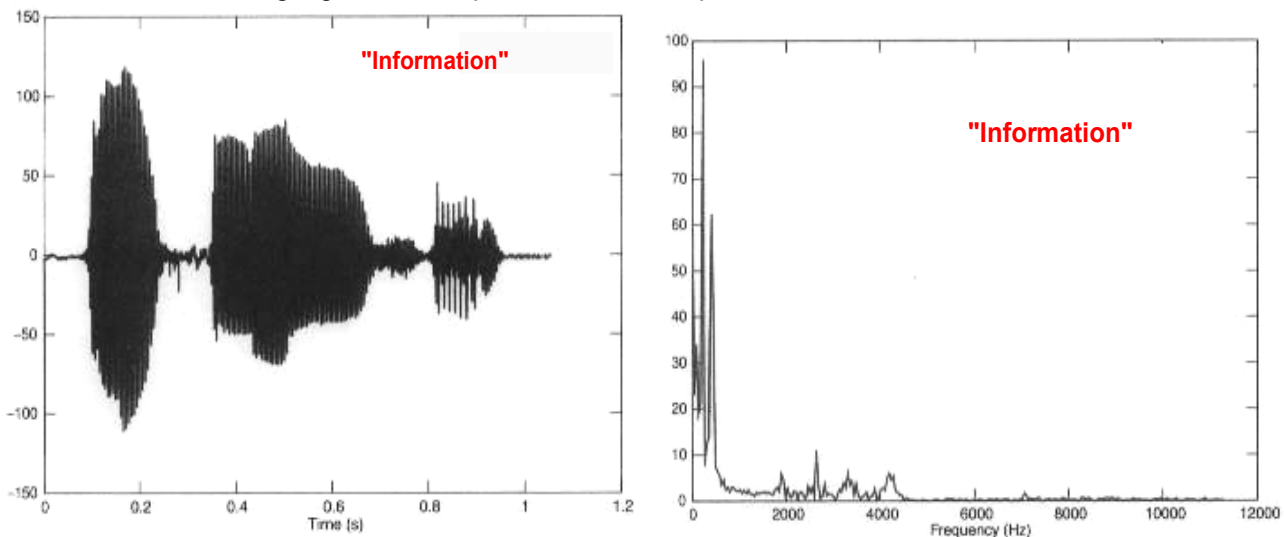


Figure 5.18 Waveform and spectrum of the word “information” uttered by a female speaker.

The idea behind pre-emphasis is to boost the low frequency components at the transmitter before modulation to artificially cause a wide range of frequencies become subject to same level of noise in the channel. On the other hand, during de-emphasis, lower frequency components are pulled back. However, the channel noise degrading the signal has not been boosted and it passes through the de-emphasis stage which attenuates the higher frequency components, where most of the noise power is concentrated. Thus, the process of pre-emphasis/de-emphasis leaves the desired signal significantly less affected by the channel noise. In order this scheme to work properly the pre-emphasis and de-emphasis filters must satisfy the inverse filtering condition:

$$H_d(\omega) = \frac{1}{H_p(\omega)} \quad (5.72)$$

The block diagrams for these pre- and post-processing stages are shown in Figure 5.19.

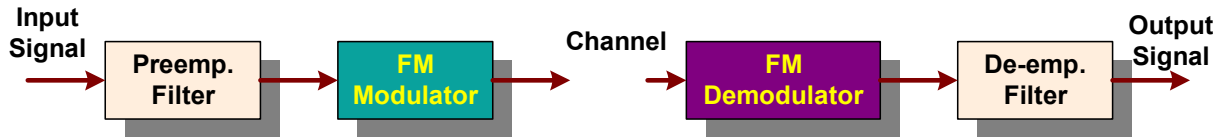


Figure 5.19 Pre-Emphasis and De-Emphasis Networks in FM Transmission.

5.8 Stereo FM Transmitters and Receivers

Federal Communications Commission (FCC) in the US as part of the ITU standard, and similarly, communications authorities in each country has allocated 88-108 MHz for FM broadcasting with a separation of 200 kHz and frequency deviation: $\Delta f = 75$ kHz for each station. Mono FM receivers operate very similar to AM superheterodyne receivers discussed in the previous chapter with the exceptions:

- (1) Intermediate frequency value $f_{IF}^{FM} = 10.7$ MHz replaces 455 kHz and
- (2) Envelope detector is replaced by a PLL or an FM discriminator followed by a de-emphasis filter with a frequency response: $H_d(w)$.

The compatible Stereo FM transmission is designed to work both with and without stereo receivers. In order to achieve this, *Left* and *Right* channel signal inputs are pre-processed in the base-band prior to any frequency translation to the FM band assigned to this broadcasting station as shown below, where L+R can be used in non-stereo receivers. Pilot tone at 19 KHz is needed to recover the original *Left* and *Right* channel signals at the receiver. The spectrum is shown in Figure 5.20.

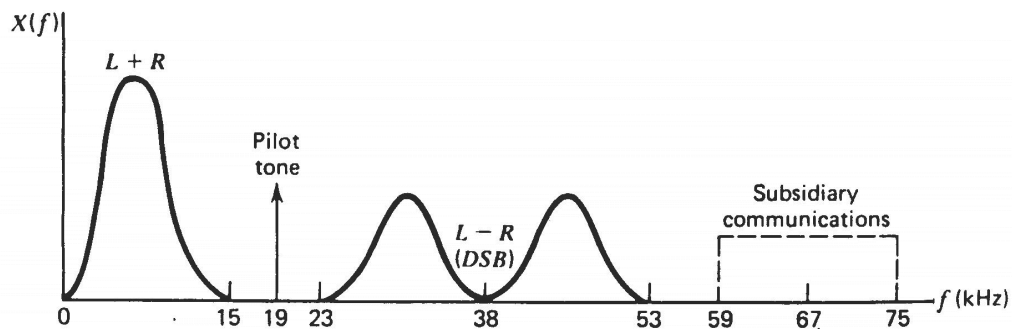


Figure 5.20 Spectrum for FM Stereo Broadcasting.

Their block diagram of an FM Stereo transmitter is shown in Figure 5.21, where “Matrix” block is used for realizing above signal mixing.

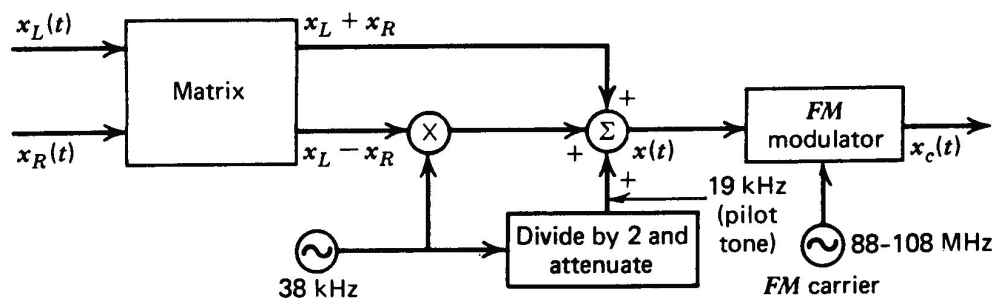


Figure 5.21 Transmitter side of a generic FM stereo broadcasting system.

The composite FM signal is created with a simple algebraic scheme followed by a pair of matched pre-emphasis filters, and a mixer output modulated by a single FM modulator.

$$x_c(t) = \left\{ (L+R) + (L-R) \cdot \cos w_p t + \alpha \cdot \cos \frac{w_p t}{2} \right\} \cdot \cos w_c t \quad (5.73)$$

Here the carrier information is injected at half the FM carrier frequency as a pilot tone $w_p / 2$ and the carrier frequency w_c is in the commercial FM band. It is not difficult to see that the receiver performs just the opposite of the blocks used in the transmitter as illustrated in Figure 5.22. In the absence of stereo receiver equipment the ordinary FM demodulator is able to regenerate the original signal.

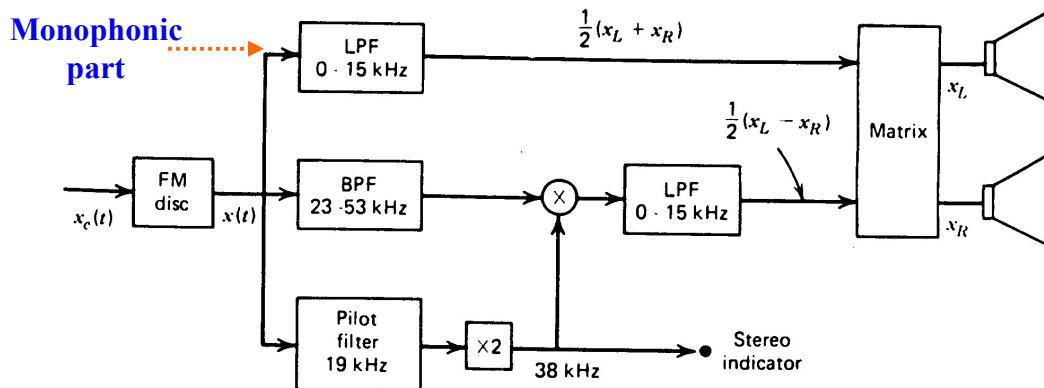


Figure 5.22 Receiver side of a generic FM stereo broadcasting system.

The receiver block diagram is very similar to the transmitter of Figure 5.22, where the pre-processing performed at the transmitter has been undone. Stereo versus mono signal is indicated by a “Stereo Indicator” signal lead in the receiver.